Lecture 8 - Carrier Drift and Diffusion *(cont.)*, Carrier Flow

February 21, 2007

Contents:

1. Quasi-Fermi levels
2. Continuity equations
3. Surface continuity equations

Reading assignment:

del Alamo, Ch. 4, §4.6; Ch. 5, §§5.1, 5.2

Key questions

- Is there something equivalent to the Fermi level that can be used outside equilibrium?
- How do carrier distributions *in energy* look like outside equilibrium?
- In the presence of carrier flow in the bulk of a semiconductor, how does one formulate bookkeeping relationships for carriers?
- How about at surfaces?

1. Quasi-Fermi levels

□ Interested in energy band diagram representations of complex situations in semiconductors *outside thermal equilibrium.*

□ In TE, Fermi level makes statement about energy distribution of carriers in bands

$\Rightarrow E_F$ relates n_o with N_c and p_o with N_v:

$$n_o = N_c \mathcal{F}_{1/2}(\frac{E_F - E_c}{kT}) \qquad p_o = N_v \mathcal{F}_{1/2}(\frac{E_v - E_F}{kT})$$

Outside TE, E_F cannot be used.

Define two "quasi-Fermi levels" such that:

$$n = N_c \mathcal{F}_{1/2}(\frac{E_{fe} - E_c}{kT}) \qquad p = N_v \mathcal{F}_{1/2}(\frac{E_v - E_{fh}}{kT})$$

Under Maxwell-Boltzmann statistics ($n \ll N_c$, $p \ll N_v$):

$$n = N_c \exp \frac{E_{fe} - E_c}{kT}$$

$$p = N_v \exp \frac{E_v - E_{fh}}{kT}$$

What are quasi-Fermi levels good for?

□ Take derivative of $n = f(E_{fe})$ with respect to x:

$$n = N_c \exp \frac{E_{fe} - E_c}{kT}$$

$$\frac{dn}{dx} = \frac{n}{kT}(\frac{dE_{fe}}{dx} - \frac{dE_c}{dx}) = \frac{n}{kT}\frac{dE_{fe}}{dx} - \frac{q}{kT}n\mathcal{E}$$

Plug into current equation:

$$J_e = q\mu_e n\mathcal{E} + qD_e\frac{dn}{dx}$$

To get:

$$J_e = \mu_e n\frac{dE_{fe}}{dx}$$

Similarly for holes:

$$J_h = \mu_h p\frac{dE_{fh}}{dx}$$

Gradient of quasi-Fermi level: *unifying driving force for carrier flow.*

□ Physical meaning of ∇E_f

For electrons,

$$J_e = \mu_e n \frac{dE_{fe}}{dx} = -qnv_e$$

Then:

$$\frac{dE_{fe}}{dx} = -\frac{q}{\mu_e} v_e$$

∇E_{fe} linearly proportional to electron velocity!

Similarly for holes:

$$\frac{dE_{fh}}{dx} = \frac{q}{\mu_h} v_h$$

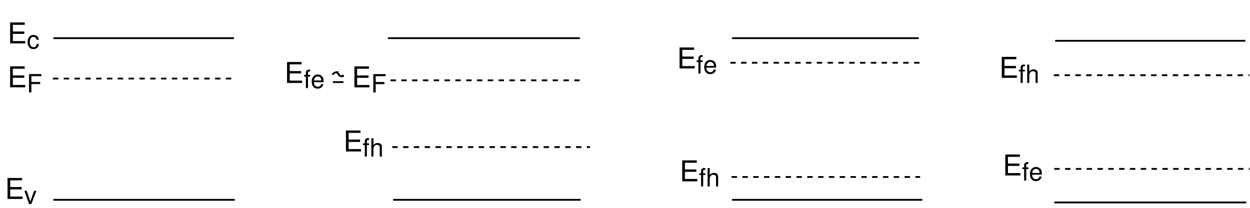
□ Quasi-Fermi levels: effective way to *visualize* carrier phenomena outside equilibrium in energy band diagram

1. Visualize carrier concentrations and net recombination

$$np = n_i^2 \exp\frac{E_{fe} - E_{fh}}{kT}$$

- If $E_{fe} > E_{fh} \Rightarrow np > n_i^2 \Rightarrow U > 0$
- If $E_{fe} < E_{fh} \Rightarrow np < n_i^2 \Rightarrow U < 0$
- If $E_{fe} = E_{fh} \Rightarrow np = n_i^2 \Rightarrow U = 0$ (carrier conc's in TE)

Examples (same semiconductor):



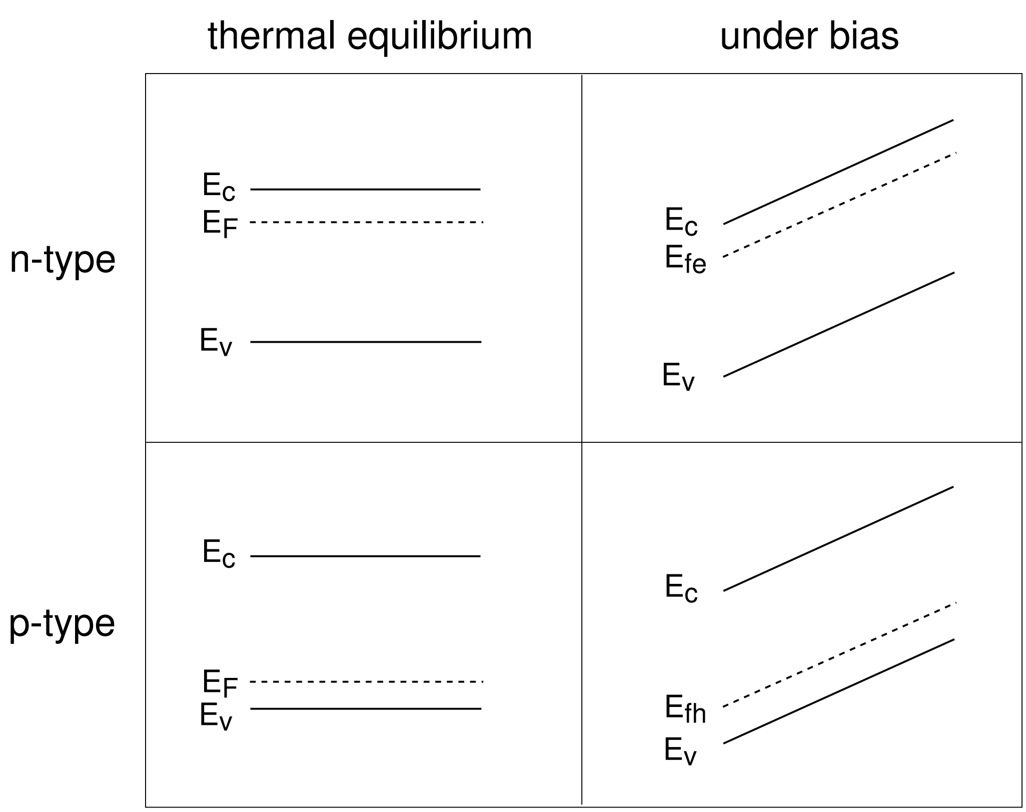
But can't visualize G_{ext}.

2. Visualize currents

$$J_e = \mu_e n \frac{dE_{fe}}{dx}$$

- $\nabla E_{fe} = 0 \;\Rightarrow\; J_e = 0$
- $\nabla E_{fe} \neq 0 \;\Rightarrow\; J_e \neq 0$
- if n high, ∇E_{fe} small to maintain a certain current level
- if n low, ∇E_{fe} large to maintain a certain current level

Examples:



□ The concept of Quasi-Fermi level hinges on notion of:

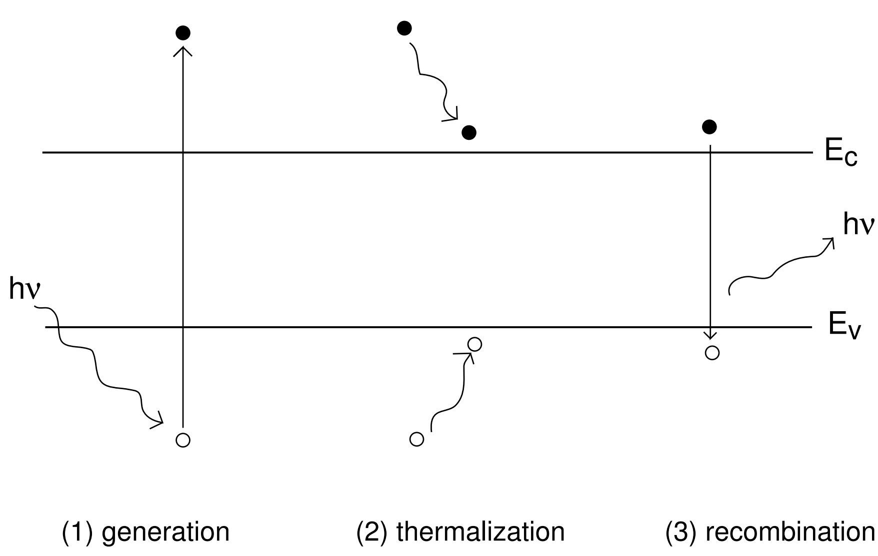
Quasi-equilibrium: *carrier distributions in energy never depart too far from TE in times scales of practical interest.*

Quasi-equilibrium appropriate if:

scattering time $\ll$ *dominant device time constant*

$\Rightarrow$ carriers undergo many collisions and attain thermal quasi-equilibrium *with the lattice and among themselves* very quickly.

In time scales of interest, carrier distribution is close to *Maxwellian* (i.e., well described by a Fermi level).



2. Continuity Equations

Semiconductor physics so far:

Gauss' law: $\vec{\nabla} \cdot \vec{\mathcal{E}} = \frac{q}{\epsilon}(p - n + N_D^+ - N_A^-)$

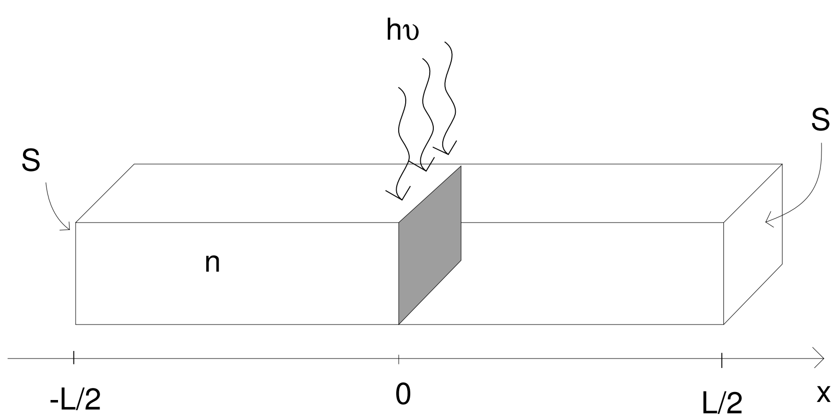
Electron current equation: $\vec{J}_e = -qn\vec{v}_e^{\,drift} + qD_e\vec{\nabla}n$

Hole current equation: $\vec{J}_h = qp\vec{v}_h^{\,drift} - qD_h\vec{\nabla}p$

Total current equation: $\vec{J}_t = \vec{J}_e + \vec{J}_h$

Carrier dynamics: $\frac{dn}{dt} = \frac{dp}{dt} = G - R$

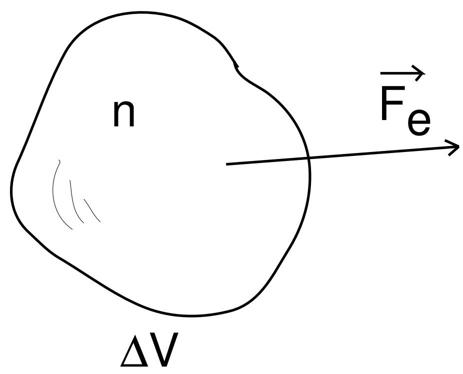
Still, can't solve problems like this:



Equation system does not capture:

- impact of carrier movement on carrier concentration (*i.e.* when carriers move away from a point, their concentration drops!)
- boundary conditions (surfaces are not infinitely far away)

Need "book-keeping relationships" for particles:



rate of increase of number of electrons in ΔV=
rate of electron generation in ΔV
- rate of electron recombination in ΔV
- net flow of electrons leaving ΔV per unit time

$$\frac{\partial(n\Delta V)}{\partial t} = G\Delta V - R\Delta V - \int \vec{F}_e.d\vec{S}$$

Dividing by ΔV and taking the limit of small ΔV:

$$\frac{\partial n}{\partial t} = G - R - \vec{\nabla}.\vec{F}_e$$

In terms of current density:

$$\frac{\partial n}{\partial t} = G - R + \frac{1}{q}\vec{\nabla}.\vec{J_e}$$

For holes:

$$\frac{\partial p}{\partial t} = G - R - \frac{1}{q}\vec{\nabla}.\vec{J_h}$$

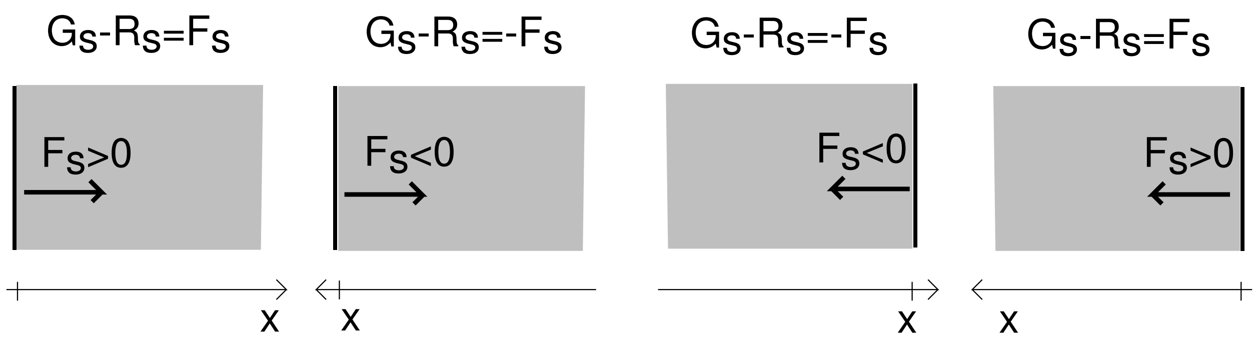
3. Surface continuity equations

□ "Free" surface: cannot "store" carriers:

Surface generation - surface recombination= carrier flow out of surface

$$|G_s - R_s| = |F_s|$$

This equation is axis sensitive.



Rewrite in terms of current densities normal to surface:

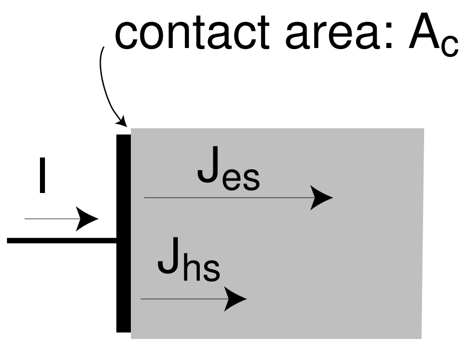
$$|U_s| = \frac{1}{q}|J_{es}| = \frac{1}{q}|J_{hs}|$$

Always, no net current into surface:

$$J_s = J_{es} + J_{hs} = 0 \Longrightarrow \; J_{es} = -J_{hs}$$

□ *Ohmic contact*: provides path for current to flow out of device.

For n-type:



Kirchoff's law demands current continuity at metal-semiconductor interface:

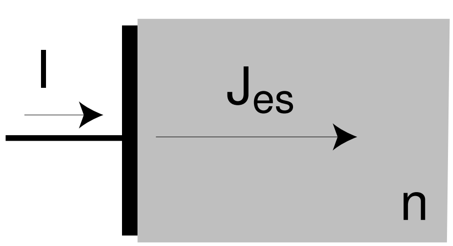
$$|I| = A_c|J_{es} + J_{hs}| = qA_c|F_{es} - F_{hs}|$$

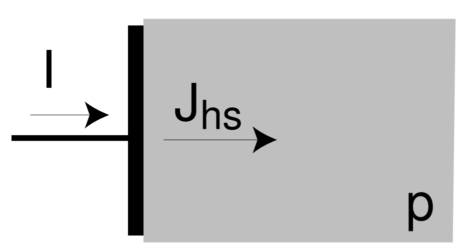
Equation is sign sensitive:

- IEEE convention: I entering into device is positive
- sign of J_{es} and J_{hs} depend of choice of axis in semiconductor

Key result from metal-semiconductor junction: Metal in ohmic contact only communicates with majority carriers in semiconductor ⇒ *majority carrier current*

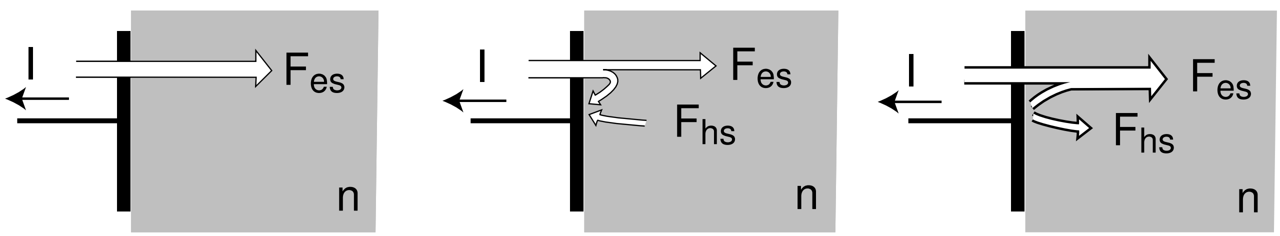
- in n-type material, current supported by electrons
- in p-type material, current supported by holes





□ If there is net generation or recombination at surface right below ohmic contact ⇒ *minority carrier current* in addition to majority carrier current

Three possible cases (examples for n-type):



ohmic contact without recombination or generation

ohmic contact with net recombination

ohmic contact with net generation

- ohmic contact with equilibrium carrier concentrations:

$$U_s = 0 \qquad |I| = qA_c|F_{es}|$$

- ohmic contact with net recombination:

$$U_s = |F_{hs}| \qquad |I| = qA_c|F_{es} - F_{hs}| > qA_c|F_{es}|$$

- ohmic contact with net generation:

$$U_s = -|F_{hs}| \qquad |I| = qA_c|F_{es} - F_{hs}| < qA_c|F_{es}|$$

Important boundary condition (will understand soon): at metal-semiconductor interface:

$$n'_s = p'_s = 0 \quad \text{or} \quad S = \infty$$

Key conclusions

- *Quasi-Fermi levels* describe carrier statistics outside equilibrium ⇒ powerful way to visualize carrier concentrations and currents in energy band diagrams.
- *Quasi-equilibrium*: carrier distributions in energy not significantly different from TE in time scales of interest for device operation.
- *Continuity equations*: "book-keeping" relations for carriers.
- Surfaces cannot store carriers: at all times must have current balance at surface.
- At "free" surface: electron and hole currents result from carrier generation or recombination at surface (but net current is zero).
- At ohmic contact:
  - additional majority carrier current to support terminal current
  - excess carrier concentrations are zero

Self-study

- Do examples 4.8, 4.9, 5.1, 5.2.
- Study integral form of continuity equations and corollary (§5.1).