6.641 Electromagnetic Fields, Forces, and Motion
Spring 2009

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Formula Sheet

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1. DIFFERENTIAL OPERATORS IN CYLINDRICAL AND SPHERICAL COORDINATES

If r, ϕ, and z are circular cylindrical coordinates and $\hat{i}_r$, $\hat{i}_\phi$, and $\hat{i}_z$ are unit vectors in the directions of increasing values of the corresponding coordinates,

$$\nabla U = \text{grad}U = \hat{i}_r \frac{\partial U}{\partial r} + \hat{i}_\phi \frac{\partial U}{\partial \phi} + \hat{i}_z \frac{\partial U}{\partial z}$$

$$\nabla \cdot \vec{A} = \text{div}\vec{A} = \frac{1}{r}\frac{\partial (rA_r)}{\partial r} + \frac{1}{r}\frac{\partial A_\phi}{\partial \phi} + \frac{\partial A_z}{\partial z}$$

$$\nabla \times \vec{A} = \text{curl}\vec{A} = \hat{i}_r \left(\frac{1}{r}\frac{\partial A_z}{\partial \phi} - \frac{\partial A_\phi}{\partial z}\right) + \hat{i}_\phi \left(\frac{\partial A_r}{\partial z} - \frac{\partial A_z}{\partial r}\right) + \hat{i}_z \left(\frac{1}{r}\frac{\partial (rA_\phi)}{\partial r} - \frac{1}{r}\frac{\partial A_r}{\partial \phi}\right)$$

$$\nabla^2 U = \text{div grad}U = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial U}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 U}{\partial \phi^2} + \frac{\partial^2 U}{\partial z^2}$$

If r, θ, and ϕ are spherical coordinates and $\hat{i}_r$, $\hat{i}_\theta$, and $\hat{i}_\phi$ are unit vectors in the directions of increasing values of the corresponding coordinates,

$$\nabla U = \text{grad}U = \hat{i}_r \frac{\partial U}{\partial r} + \hat{i}_\theta \frac{1}{r}\frac{\partial U}{\partial \theta} + \hat{i}_\phi \frac{1}{r\sin\theta}\frac{\partial U}{\partial \phi}$$

$$\nabla \cdot \vec{A} = \text{div}\vec{A} = \frac{1}{r^2}\frac{\partial (r^2 A_r)}{\partial r} + \frac{1}{r\sin\theta}\frac{\partial (A_\theta \sin\theta)}{\partial \theta} + \frac{1}{r\sin\theta}\frac{\partial A_\phi}{\partial \phi}$$

$$\nabla\times\vec{A} = \text{curl}\vec{A} = \hat{i}_r \left(\frac{1}{r\sin\theta}\frac{\partial (A_\phi \sin\theta)}{\partial \theta} - \frac{1}{r\sin\theta}\frac{\partial A_\theta}{\partial \phi}\right) + \hat{i}_\theta \left(\frac{1}{r\sin\theta}\frac{\partial A_r}{\partial \phi} - \frac{1}{r}\frac{\partial (rA_\phi)}{\partial r}\right) + \hat{i}_\phi \left(\frac{1}{r}\frac{\partial (rA_\theta)}{\partial r} - \frac{1}{r}\frac{\partial A_r}{\partial \theta}\right)$$

$$\nabla^2 U = \text{div grad}U = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial U}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial U}{\partial \theta}\right) + \frac{!}{r^2\sin^2\theta}\frac{\partial^2 U}{\partial \phi^2}$$

2. SOLUTIONS OF LAPLACE'S EQUATIONS

A. Rectangular coordinates, two dimensions (independent of z):

$$\Phi = e^{kx}(A_1 \sin ky + A_2 \cos ky) + e^{-kx}(B_1 \sin ky + B_2 \cos ky)$$

(or replace e^{kx} and e^{-kx} by $\sinh kx$ and $\cosh kx$)

$$\Phi = Axy + Bx + Cy + D; \ (k = 0)$$

B. Cylindrical coordinates, two dimensions (independent of z):

$$\Phi = r^n(A_1 \sin n\phi + A_2 \cos n\phi) + r^{-n}(B_1 \sin n\phi + B_2 \cos n\phi)$$

$$\Phi = \ln\frac{R}{r}(A_1\phi + A_2) + B_1\phi + B_2; \ (n = 0)$$

C. Spherical coordinates, two dimensions (independent of ϕ):

$$\Phi = Ar\cos\theta + \frac{B}{r^2}\cos\theta + \frac{C}{r} + D$$