

6.641 Electromagnetic Fields, Forces, and Motion  
Spring 2009

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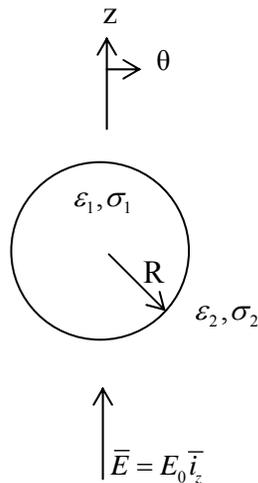
Massachusetts Institute of Technology  
 Department of Electrical Engineering and Computer Science  
 6.641 Electromagnetic Fields, Forces, and Motion

Quiz 2  
 May 3, 2006

6.641 Formula Sheet appears at the end of this quiz. You are also allowed to bring **two** 8 1/2" x 11" sheet of notes (both sides) that **you** prepare.

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Problem 1 (50 points)



See figure from 5P.11 from *Electromechanical Dynamics*, by Herbert H. Woodson and James R. Melcher.  
 A constant electric field is applied at  $r = \infty$

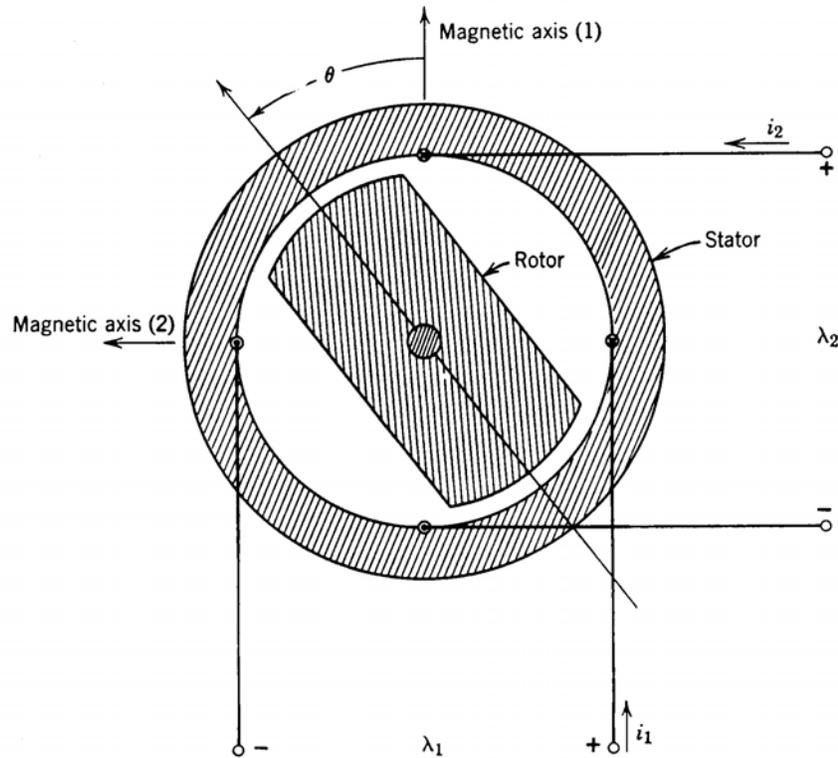
$$\lim_{r \rightarrow \infty} \vec{E} = E_0 \vec{i}_z$$

which is incident upon a sphere of radius  $R$  whose center is located at  $z = 0$ . The sphere, with dielectric permittivity  $\epsilon_1$  and ohmic conductivity  $\sigma_1$ , is placed within a medium with dielectric permittivity  $\epsilon_2$  and ohmic conductivity  $\sigma_2$ . The system is in the time independent steady state.

- a) What are the necessary boundary conditions to solve for the electrostatic scalar potential  $\Phi(r, \theta)$  and electric field  $\vec{E}(r, \theta)$  inside and outside the sphere in the time independent steady state.
- b) Find  $\Phi(r, \theta)$  and  $\vec{E}(r, \theta)$  in the time independent steady state.
- c) What is the free surface charge distribution on the  $r = R$  interface?
- d) For what relationship between  $\epsilon_1, \epsilon_2, \sigma_1$  and  $\sigma_2$  is the free surface charge density zero for all  $\theta$  on the  $r = R$  interface?
- e) What is the effective dipole moment of the sphere for fields in the region  $r > R$ ?

Problem 2 (50 points)

The figure below shows a diagrammatic cross section of a two-phase, salient-pole synchronous machine. The windings in an actual machine are distributed in many slots



See figure from 5P.11 from *Electromechanical Dynamics*, by Herbert H. Woodson and James R. Melcher.

along the periphery of the stator, rather than as shown. The rotor is made of magnetically soft iron which has no residual permanent magnetism. The electrical terminal relations are given by

$$i_1 = \frac{\lambda_1 [L_0 - M \cos 2\theta] - \lambda_2 M \sin 2\theta}{L_0^2 - M^2}$$

$$i_2 = \frac{-\lambda_1 M \sin 2\theta + \lambda_2 (L_0 + M \cos 2\theta)}{L_0^2 - M^2}$$

where  $L_0$  and  $M$  are self and mutual inductances independent of  $\theta$  with  $L_0 > M$ .

a) Determine the magnetoquasistatic torque  $T^M(\lambda_1, \lambda_2, \theta)$ .

b) Assume that the machine is excited by voltage sources such that  $V_1 = \frac{d\lambda_1}{dt} = V_0 \cos \omega t$ ,

$V_2 = \frac{d\lambda_2}{dt} = V_0 \sin \omega t$ , and the rotor has the constant angular velocity  $\omega_m$  such that  $\theta = \omega_m t + \gamma$ .

What are  $\lambda_1$  and  $\lambda_2$  as a sinusoidal steady state function of time? Evaluate the instantaneous torque  $T^M$ . Under what conditions is it constant?

- c) The rotor is subject to a mechanical torque (acting on it in the  $+\theta$ -direction):  $T = T_0 + T'(t)$ , where  $T_0$  is a positive constant. The time-varying part of the torque perturbs the steady rotation of (b) so that  $\theta = \omega_m t + \gamma_0 + \gamma'(t)$ . Assume that the rotor has a moment of inertia  $J$  but that there is no damping. Find the possible equilibrium angles  $\gamma_0$  between the rotor and the stator field and indicate which are stable and unstable. Then write a differential equation for  $\gamma'(t)$ , with  $T'(t)$  as a driving function.
- d) Consider small perturbations of the rotation  $\gamma'(t)$ , so that the equation of motion found in (c) can be linearized. Find the response to an impulse of torque  $T'(t) = I_0 \delta(t)$ , assuming that before the impulse in torque the rotation velocity is constant at  $\omega_m$ .
- e) Which of the equilibrium phase angles  $\gamma_0$  found in (c) is stable? Verify the stability or instability of the equilibrium angles found in part (c) using the results of part (d).