6.641 Electromagnetic Fields, Forces, and Motion
Spring 2009

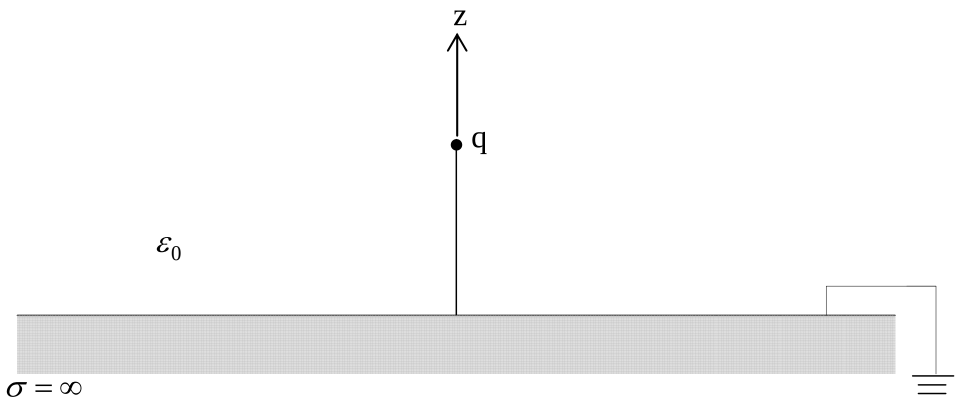
Massachusetts Institute of Technology
Department of Electrical Engineering and Computer Science
6.641 Electromagnetic Fields, Forces, and Motion

Quiz 1

March 22, 2006

6.641 Formula Sheet appears at the end of this quiz. You are also allowed to bring one 8 ½” x 11” sheet of notes (both sides) that **you** prepare.

1.



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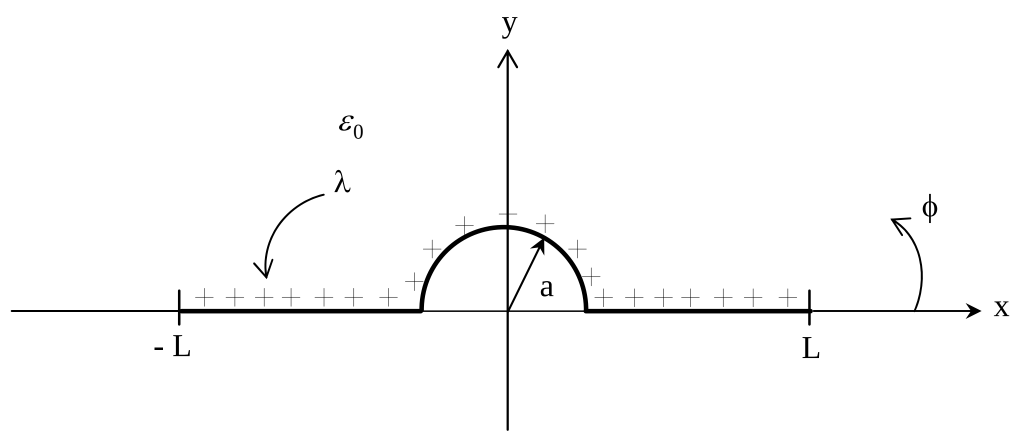
A charge q of mass m in free space is above a perfectly conducting (σ=∞) ground plane for z<0. The charge is released from rest at position z=d at t=0. Neglect the effects of gravity.

(a) What is the velocity of the charge as a function of position z?
(b) How long does it take the charge to reach the z=0 ground plane?

Hint: $\int \frac{dz}{[\frac{1}{z}-\frac{1}{d}]^{1/2}} = -\sqrt{zd(d-z)} + d^{3/2}\tan^{-1}\sqrt{\frac{z}{d-z}}$

2.



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A uniformly distributed line charge λ in the z=0 plane extends from – L to L composed of two straight sections, -L<x<-a and a<x<L, and a semi-circular section of radius a. The line charge is within free space. The potential and electric field for any line charge distribution is

$$\Phi(\bar{r}) = \int_{l'} \frac{\lambda(\bar{r}')\, dl'}{4\pi\varepsilon_0 \left|\bar{r} - \bar{r}'\right|}$$

$$\bar{E}(\bar{r}) = \int_{l'} \frac{\lambda(\bar{r}')\bar{i}_{r'r}\ dl'}{4\pi\varepsilon_0 \left|\bar{r} - \bar{r}'\right|^2}$$

(a) Find the potential at the point (x=0, y=0, z=0).
(b) Find the electric field (magnitude and direction) at (x=0, y=0, z=0).

3.

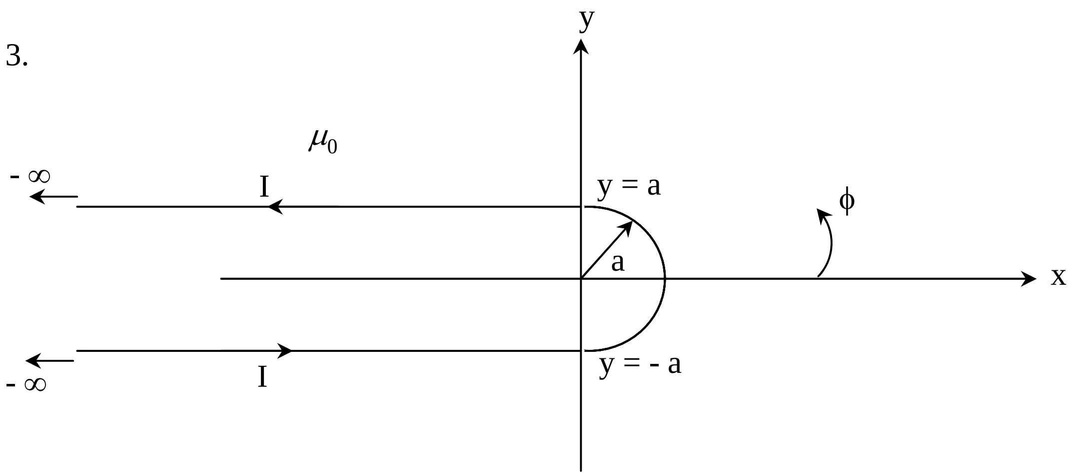

Image by MIT OpenCourseWare.

A line current in free space in the z=0 plane and carrying a current I is shaped like a “hairpin”, composed of two straight sections of semi-infinite length a distance 2a apart, joined by a semi-circular section of radius a. The magnetic field from a line current is given by the Biot-Savart law:

$$\overline{H}(\overline{r}) = \frac{1}{4\pi}\int \frac{\overline{I}dl' \times \overline{i}_{r'r}}{\left|\overline{r}-\overline{r}'\right|^2}$$

What is the magnetic field $\overline{H}$ at the point (x=0, y=0, z=0)?

Hint: one or more of the following indefinite integrals may be useful:

a) $$\int \frac{dx}{[x^2+a^2]^{1/2}} = \ln[x+\sqrt{x^2+a^2}]$$

b) $$\int \frac{xdx}{[x^2+a^2]^{1/2}} = [x^2+a^2]^{1/2}$$

c) $$\int \frac{dx}{[x^2+a^2]} = \frac{1}{a}\tan^{-1}\frac{x}{a}$$

d) $$\int \frac{dx}{[x^2+a^2]^{3/2}} = \frac{x}{a^2[x^2+a^2]^{1/2}}$$

e) $$\int \frac{xdx}{[x^2+a^2]^{3/2}} = -\frac{1}{[x^2+a^2]^{1/2}}$$