6.641 Electromagnetic Fields, Forces, and Motion
Spring 2005

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Problem Set 8 - Questions

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Problem 8.1 (W&M Prob 6.1)

Two frames of reference have a relative angular velocity Ω, as shown in Fig. 6P.1. In the fixed frame a point in space is designated by the cylindrical coordinates (r, θ, z).In the rotating frame the same point is designated by $(r^{'}, \theta^{'}, z^{'})$. Assume that $t = t^{'}$.

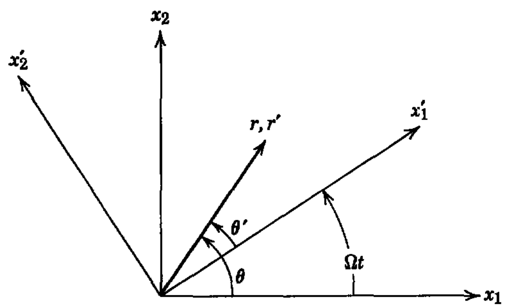

Fig. 6P.1

Figure 1: Two frames of reference with relative angular velocity Ω

A

Write the transformation laws [like (6.1.6)] that relate primed coordinates to the unprimed coordinates.

B

Given that ψ is a function of (r, θ, z, t), find $\frac{\partial \psi}{\partial t'}$ (the rate of change with respect to time of ψ for an observer in the rotating frame) in terms of derivatives with respect to (r, θ, z, t).

Woodson, Herbert H., and James R. Melcher. *Electromechanical Dynamics, Part 1: Discrete Systems*. Malabar, FL: Kreiger Publishing Company, 1968. ISBN: 9780894644597.

Problem 8.2 (W&M Prob 6.6)

A pair of cylinders coaxial with the z-axis, as shown in Fig. 6P.6, forms a capacitor. The inner and outer surfaces have the potential difference V and radii a and b, respectively. The cylinders are only very slightly conducting, so that as they rotate with the angular velocity ω they carry along the charges induced on their

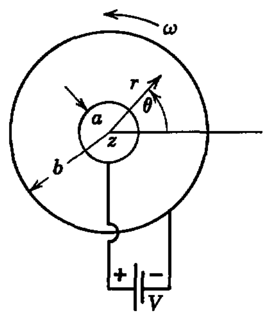

Fig. 6P.6

Figure 2: A pair of cylinders coaxial with the z-axis

surfaces. As viewed from a frame rotating with the cylinders, the charges are stationary. We wish to compute the resulting fields.

A

Compute the electric field between the cylinders and the surface charge densities σ_a and σ_b on the inner and outer cylinders, respectively.

B

Use the transformation for the current density to compute the current density from the results of part (a).

C

In turn, use the current density to compute the magnetic field intensity $\mathbf{H}$ between the cylinders.

D

Now use the field transformation for the magnetic field intensity to check the result of part (c).

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Problem 8.3 (W&M Prob 6.12)

The system shown in Fig. 6P.12 consists of two parallel, perfectly conducting plates with depth D and separation W. Between these plates is placed a perfectly conducting short-circuit which has mass M and slides with viscous coefficient of friction B. You may assume $W \ll D$ and that fringing fields may be neglected.

A

Find $\lambda = \lambda(i, x)$.

B

Find $W_m'(i,x)$ or $W_m(\lambda,x)$.

C

Find the force of electric origin f^e (from $W_m'(i,x)$ or $W_m(\lambda,x)$) exerted by the fields on the sliding short.

Assume now that a battery is placed across the electric terminals so that $v = V_0 =$ constant.

D

Write a complete set of differential equations that would allow you to find $x(t)$.

E

If the system has reached a state in which the velocity of the plate $(\frac{dx}{dt})$ is a constant, find $(\frac{dx}{dt})$.

F

Under the conditions of (e), find the current supplied by the battery $i(t)$.

You will now repeat this problem and solve it by using field theory. Do not assume that $v = V_0$ and $x=$ constant until part (l).

G

Find the magnetic field **H** between the plates as a function of the current $i = i(t)$.

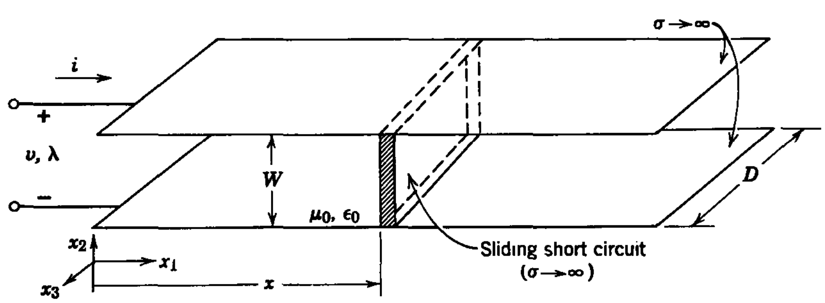

Fig. 6P.12

Figure 3: Two parallel, perfectly conducting plates with a sliding short circuit

H

Find the force exerted by this **H** field on the sliding short in terms of i and x by using the Lorentz force law to show that it agrees with (c).

I

Compute the electric field everywhere between the plates. Evaluate the constant of integration by requiring that the voltage at the terminals be $v(t)$.

J

Relate the terminal voltage $v(t)$ to the current $i(t)$ and plate position $x(t)$ by explicitly using Faraday's law in integral form.

K

Show by using (i) and (j) that the boundary conditions on the electric field at the moving plate are satisfied.

L

Convince yourself that the results of (g) through (k) are formally equivalent to the lumped-parameter approach of parts (a) through (f); that is, again find $(\frac{dx}{dt})$ and $i(t)$ by assuming that $v = V_0$ and $(\frac{dx}{dt}) =$ constant.

M

Under the conditions of part (l) evaluate the electric field of part (i) explicitly.

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Problem 8.4 (W&M Prob 6.14)

Figure 4 (6P.14) shows a model for a self-excited dc machine. The rotor is laminated in such a way that i flows only in the z direction. The brushes have an effective area A, hence the current density on either side of the rotor is $\frac{i}{A}$. The rotor has conductivity σ over the area of the brushes. Neglect the thickness of the brushes compared with r. The far end of the rotor is assumed to be infinitely conducting, as shown. The rotor is driven at a constant angular velocity ω; R_{int} does not include the effect of the conductivity of the rotor.

A

Find the differential equation for $i(t)$. Neglect the inductance of the rotor.

B

If $i(t = 0) = I_0$, calculate the power dissipated in the load resistor R_L as a function of time. For what values of the parameters is this power unbounded as $t \to \infty$?

C

In a real system what would prevent the current from becoming infinite?

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Problem 8.5 (W&M Prob 6.20)

A dc machine is often used as an energy storage element. With constant field current, negligible mechanical losses, and negligible armature inductance, the machine, as viewed from the armature terminals, appears as the RC circuit in Fig. 5 (6P.20).

A

Find the equivalent capacitance C in terms of G, I_f, and J_r.

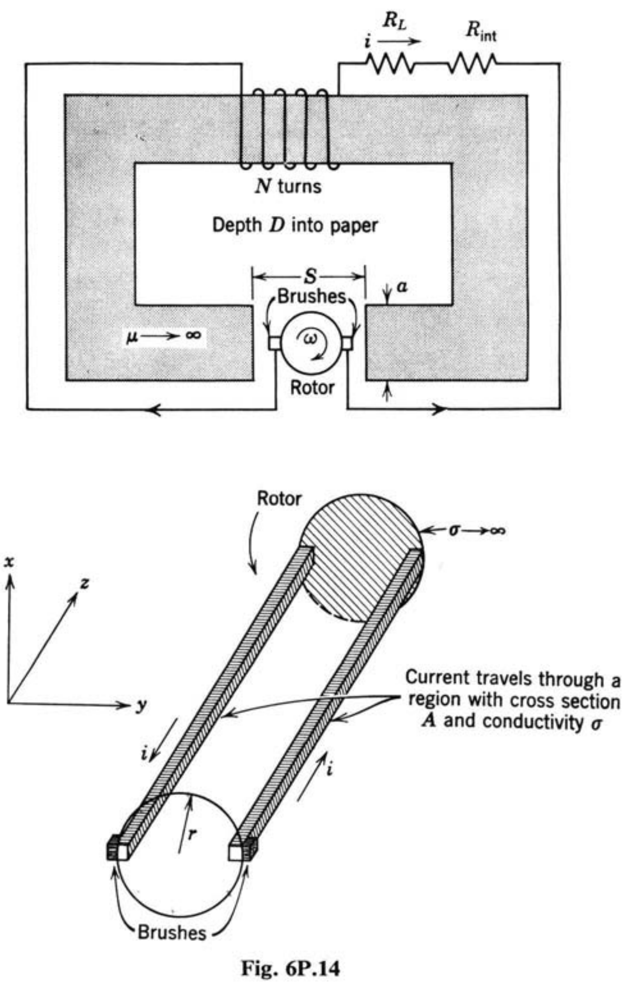

Figure 4: A model for a self-excited dc machine

B

Evaluate C for the machine in Problem 6.19 with $I_f = 1$ A.

Problem 8.6 (Zahn Prob 6.21, pp. 477-78)

The field winding of a homopolar generator is connected in series with the rotor terminals through a capacitor C. The rotor is turned at a constant speed ω. See Fig. 6.

A

For what minimum value of rotor speed is the system self-excited?

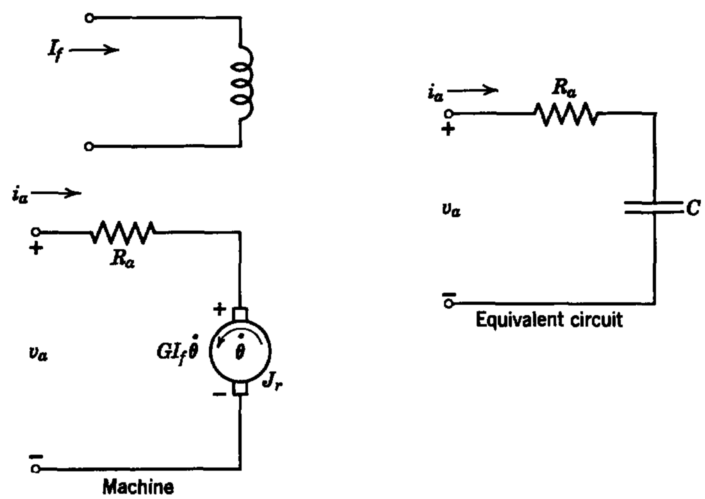

Fig. 6P.20

Figure 5: RC equivalent circuit for a homopolar generator.

B

For the self-excited condition of (a) what range of values of C will result in dc self-excitation or in ac self-excitation?

C

What is the frequency for ac self-excitation?

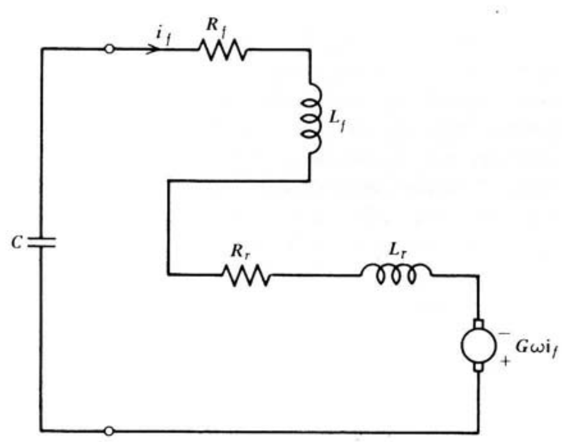

Figure 6: A self-excited homopolar generator with field and armature windings in series with a capacitor.