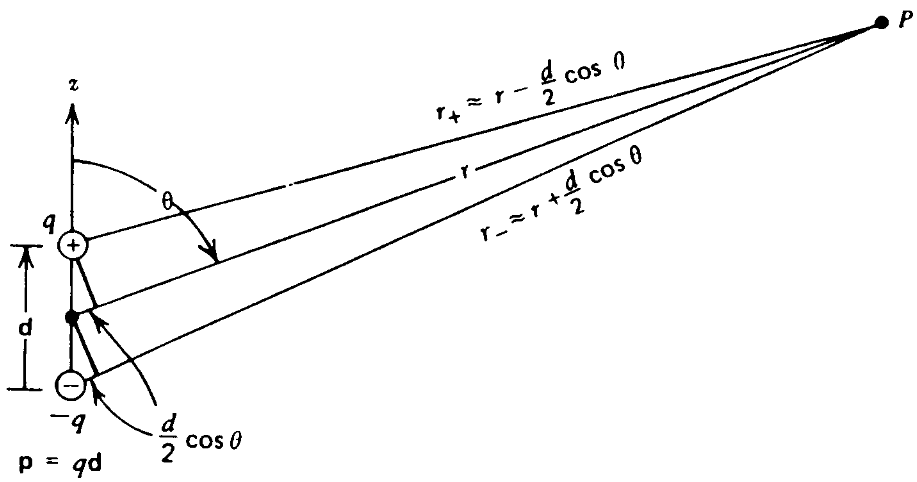
Massachusetts Institute of Technology
Department of Electrical Engineering and Computer Science
6.641 Electromagnetic Fields, Forces, and Motion

Problem Set #3 — Issued: 2/15/05
Spring Term 2005 — Due: 2/24/05

Suggested Reading Assignment: Zahn – 3.1.1, 5.2.1, 5.7.1
Suggested Video Viewing: H/M Demos 1.4.1, 1.6.1, 4.7.1
[http://web.mit.edu/6.013_book/www/VideoDemo.html]
Demos: 8.2.1, 8.2.2

Problem 3.1

An electric dipole consists of two opposite polarity charges, $\pm q$ at $z = \pm d/2$.



(a) Start with the electric potential of a point charge, and determine $\Phi(r,\theta)$ for the electric dipole.

(b) Define the dipole moment as $p=qd$ and show that in the limit where $d \to 0$ (while p remains finite), the electric potential is

$$\Phi(r,\theta) = \frac{p}{4\pi\varepsilon_o}\frac{\cos\theta}{r^2}$$

(c) What is the electric field for the dipole of part (b) with $d \to 0$ with p remaining finite?

(d) The electric field lines are lines that are tangent to the electric field:

$$\frac{dr}{r d\theta} = \frac{E_r}{E_\theta}$$

Using the result of (c), integrate this equation to find the field line that passes through the radial point r_0 when $\theta = \pi/2$. This analytical equation can be used to precisely plot the electric field lines.

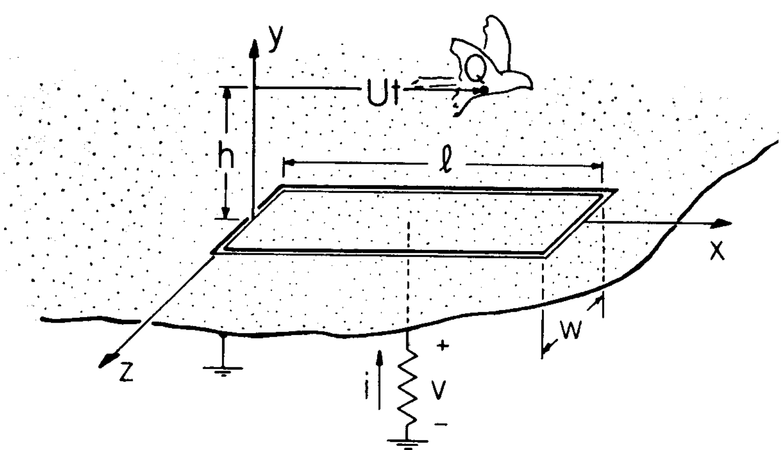
Hint: $\int cot\theta d\theta = \ln(\sin\theta) + \text{constant}$

(e) Use your favorite computer plotting routine to plot on the same plot the equipotential and electric field lines for $4\pi\varepsilon_o / p = 100$ volt^{-1}-m^{-2}. Draw electric field lines for r_0=0.25, 0.5, 1 and 2 meters and draw equipotential lines for $\Phi = 0, \pm.0025, \pm.01, \pm.04, \pm.16$ and $\pm.64$ volts.

Problem 3.2

When a bird perches on a dc high-voltage power line and then flies away, it does so carrying a net charge.

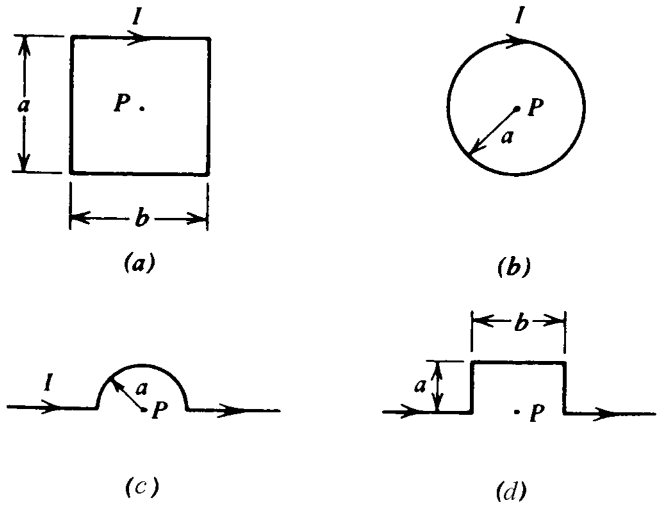
(a) Why?
(b) For the purpose of measuring this net charge Q carried by the bird, we have the apparatus pictured below. Flush with the ground, a strip electrode having width w and length l is mounted so that it is insulated from ground. The resistance, R, connecting the electrode to ground is small enough that the potential of the electrode (like that of the surrounding ground) can be approximated as zero. The bird flies in the x direction at a height h above the ground with a velocity U. Thus, its position is taken as $y=h$ and $x=Ut$. At time t, what is the effective charge distribution that will allow easy calculation of the electric scalar potential?
(c) The bird flies at an altitude h sufficiently large to make it appear as a point charge. What is the potential distribution as a function of time and position (x, y, z)?
(d) Determine the surface charge density $\sigma_s(x, y=0, z, t)$ on the ground plane at $y=0$ as a function of time.
(e) At time t, what is the net charge, q, on the electrode? (Assume that the width w is small compared to h so that in an integration over the electrode surface, the integration in the z direction is simply a multiplication by w.)
Hint: Let $x' = x - Ut$
Hint: $\int \frac{dx}{[a^2 + x^2]^{3/2}} = \frac{x}{a^2[a^2 + x^2]^{1/2}}$
(f) The current through the resistor is dq/dt. Find an expression for the voltage, v, that would be measured across the resistance, R.



"Bird on Powerline" diagram from: *Electromagnetic Fields and Energy* by Hermann A. Haus and James R. Melcher.

Problem 3.3

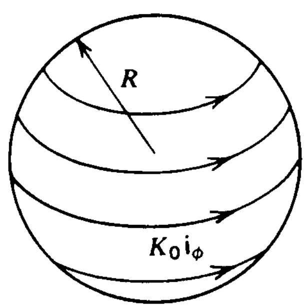
Find the magnetic field intensity at the point *P* shown due to the following line currents:



"Electric Dipole", "Line Current", "Sphere of Radius" and "Perfectly Conducting Plane" diagrams from: *Electromagnetic Field Theory: A Problem Solving Approach*, by Markus Zahn, Robert E. Krieger Publishing Company, 1987. Used with permission.

Problem 3.4

A constant current $K_o \bar{i}_\phi$ flows on the surface of a sphere of radius *R*.

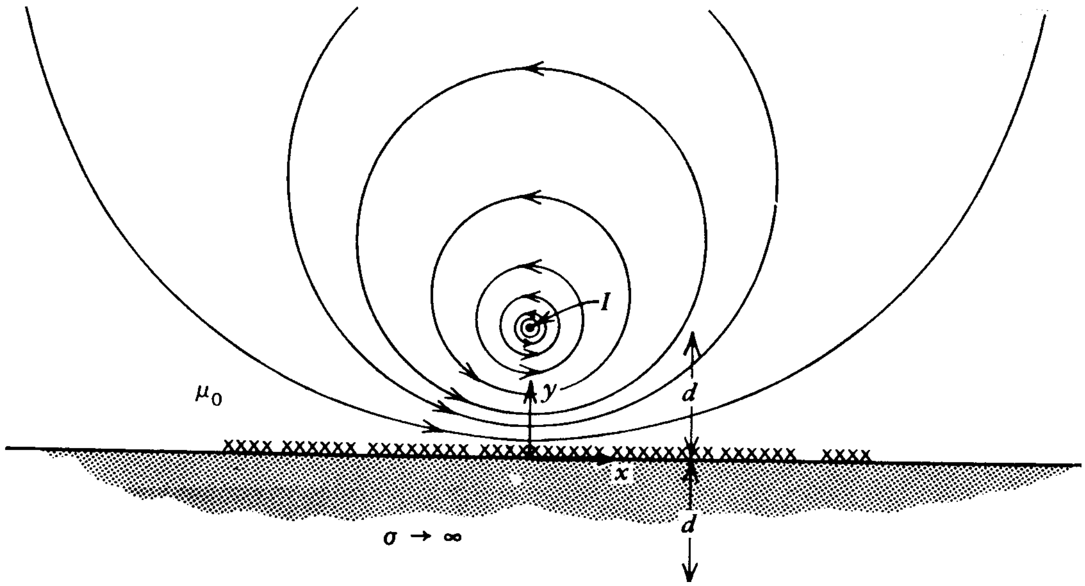


"Electric Dipole", "Line Current", "Sphere of Radius" and "Perfectly Conducting Plane" diagrams from: *Electromagnetic Field Theory: A Problem Solving Approach*, by Markus Zahn, Robert E. Krieger Publishing Company, 1987. Used with permission.

(a) What is the magnetic field intensity at the center of the sphere?
(Hint: $\bar{i}_\phi \times \bar{i}_r = \cos\theta\cos\phi\,\bar{i}_x + \cos\theta\sin\phi\,\bar{i}_y - \sin\theta\,\bar{i}_z$)

(b) Use the results of (a) to find the magnetic field intensity at the center of a spherical shell of inner radius R_1 and outer radius R_2 carrying a uniformly distributed volume current $\vec{J} = J_o \bar{i}_\phi$.

Problem 3.5



"Electric Dipole", "Line Current", "Sphere of Radius" and "Perfectly Conducting Plane" diagrams from: *Electromagnetic Field Theory: A Problem Solving Approach*, by Markus Zahn, Robert E. Krieger Publishing Company, 1987. Used with permission.

A line current *I* of infinite extent in the z-direction is at a distance d above a perfectly conducting plane.

(a) Use the method of images to satisfy boundary conditions and find the magnetic vector potential for $y>0$.
(b) What is the magnetic field for $y>0$?
(c) What is the surface current distribution that flows on the $y=0$ surface?
(d) What is the force per unit length on the line current at $y=d$?