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6.641 Electromagnetic Fields, Forces, and Motion
Spring 2005

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| **6.641 — Electromagnetic Fields, Forces, and Motion** | Spring 2005 |
| Problem Set 11 - Questions | |
| Prof. Markus Zahn | MIT OpenCourseWare |

# Problem 11.1 (W&M Prob 10.8)

An electromagnetic wave can be transmitted through or reflected by a plasma, depending on the frequency of the wave relative to the plasma frequency $\omega_p$. This phenomenon, which is fundamental to the propagation of radio signals in the ionosphere, is illustrated by the following simple example of a cutoff wave. In dealing with electromagnetic waves, we require that both the electric displacement current in Ampere's law and the magnetic induction in Faraday's law (See Section B.2.1) be accounted for. We consider one-dimensional plane waves in which $\mathbf{E} = \mathbf{i_x} E_x(z,t)$ and $\mathbf{H} = \mathbf{i_y} H_y(z,t)$.

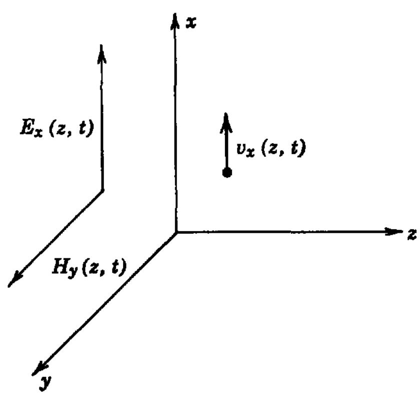

**Fig. 10P.8**

Figure 1: An electromagnetic wave incident onto a plasma.

## A

Show that Maxwell's equations require that

$$\frac{\partial E_x}{\partial z} = -\frac{\partial \mu_0 H_y}{\partial t}, \qquad -\frac{\partial H_y}{\partial z} = \frac{\partial \varepsilon_0 E_x}{\partial t} + J_x$$

.

## B

The space is filled with plasma composed of equal numbers of ions and electrons. Assume that the more massive ions remain fixed and that $n_e$ is the electron number density, whereas $e$ and $m$ are the electronic charge and mass. Use a linearized force equation to relate $E_x$ and $v_x$, where $v_x$ is the average electron velocity in the $x$-direction.

## C

Relate $v_x$ and $J_x$ to linear terms.

### D

Use the equations of (a)-(c) to find the dispersion equation for waves in the form of $e^{j(\omega t - kz)}$.

### E

Define the plasma frequency as $\omega_p = \sqrt{\frac{n_e e^2}{\varepsilon_0 m}}$ and describe the dynamics of a wave with $\omega < \omega_p$.

### F

Suppose that a wave in free space were to be normally incident on a layer of plasma (such as the ionosphere). What would you expect to happen? (See Problem 10.9 for a similar situation.)

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Woodson, Herbert H., and James R. Melcher. *Electromechanical Dynamics, Part 2: Fields, Forces, and Motion*. Malabar, FL: Kreiger Publishing Company, 1968. ISBN: 9780894644597.

## Problem 11.2 (W&M Prob 10.10)

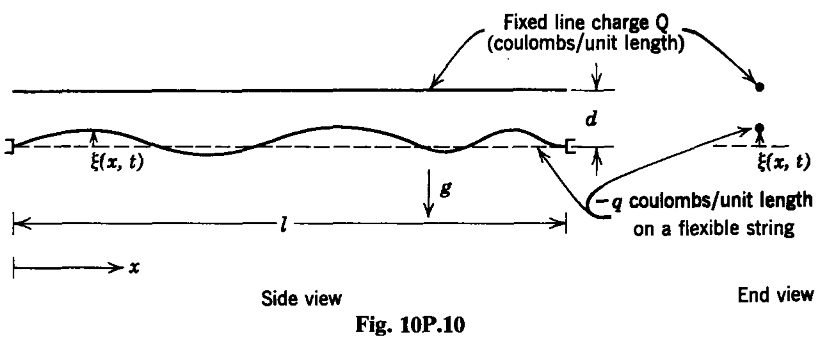


Figure 2: A charged insulating string is below an oppositely charged rigid rod.

A rigid straight rod supports a charge $Q$ coulombs per unit length and is fixed. Just below this rod an insulating string is stretched between fixed supports, as shown in Fig. 10P.10. This string, which has a tension $f$ and mass per unit length $m$, supports a charge per unit length $-q$, where $q \ll Q$ and both $Q$ and $q$ are positive.

### A

What should $qQ$ be in order that the string have the static equilibrium $\xi = 0$ in spite of the gravitational acceleration $g$?

### B

What is the largest value of $m$ that is consistent with the equilibrium of part (a) being stable?

### C

How would you alter this physical situation to make the static equilibrium stable even with $m$ larger than given by (b)?

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# Problem 11.3 (W&M Prob 10.13)

A pair of perfectly conducting membranes are stretched between rigid supports at $x = 0$ and $x = L$, as shown in FIg. 10P.13. The membranes have the applied voltage $V_\theta$ with respect to each other and with respect to plane-parallel electrodes.

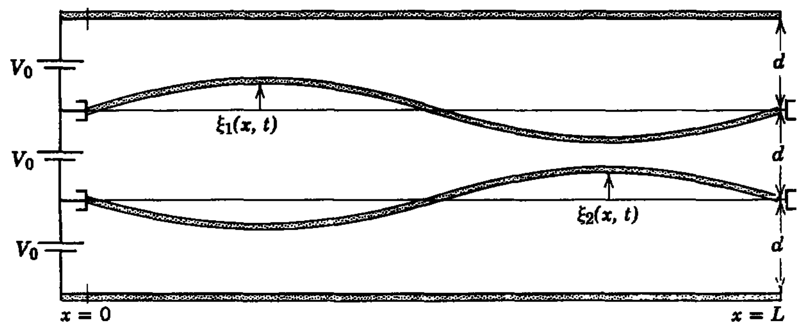


**Fig. 10P.13**

Figure 3: A pair of perfectly conducting membranes at voltage differences $V_0$ with respect to each other and with respect to plane-parallel electrodes.

### A

Write a pair of differential equations in $\xi_1(x, t)$ and $\xi_2(x, t)$ which describe the membrane motions. Assume that $\xi_1$ and $\xi_2$ are small enough to warrant linearization and that the wavelengths are long enough that the membranes appear flat to the electric field at any one value of $x$.

### B

Assume that

$$\xi_1 = \text{Re}\hat{\xi}_1 e^{j(\omega t - kx)}$$
$$\xi_2 = \text{Re}\hat{\xi}_2 e^{j(\omega t - kx)}$$

and find a dispersion equation relating $\omega$ and $k$.

### C

Make an $\omega - k$ plot showing the results of part (b), including imaginary values of $\omega$ for real values of $k$. (This equation should be biquadratic in $\omega$).

### D

At what potential $V_0$ will the static equilibrium $\xi_1 = 0$ and $\xi_2 = 0$ first become unstable? Describe the mode of instability.

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# Problem 11.4 (W&M Prob 10.23)

A pair of perfectly conducting membranes move in the $x$-direction with the velocity $U$. The membranes have the applied voltage $V_0$ with respect to one another and to plane-parallel electrodes. They enter the region between the plates from rollers at $x = 0$.

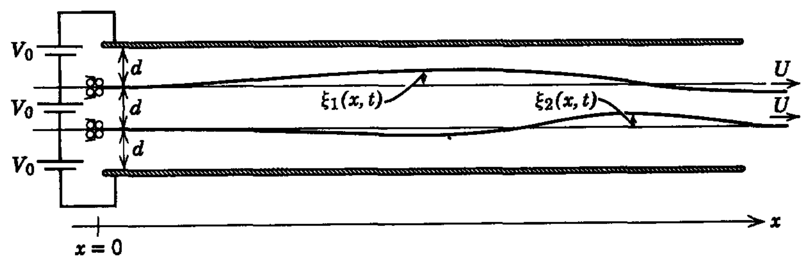


Figure 4: A pair of perfectly conducting membranes at respective voltages $V_0$ moving with velocity $U$.

## A

Write a pair of differential equations in $\xi_1(x,t)$ and $\xi_2(x,t)$ to describe the membrane motions. Assume that $\xi_1$ and $\xi_2$ are small enough to warrant linearization and that the wavelengths are long enough for the membrane to appear flat to the electric field at any one value of $x$.

## B

Assume that

$$\xi_1 = \text{Re}\hat{\xi}_1 e^{j(\omega t - kx)}$$
$$\xi_2 = \text{Re}\hat{\xi}_2 e^{j(\omega t - kx)}$$

and find a dispersion equation relating $\omega$ and $k$.

## C

Make an $\omega - k$ plot to show the results of part (b), including complex values of $k$ for real values of $\omega$. This equation can be factored into two quadratic equations for $k$. Assume that $U > \sqrt{S/\sigma_m}$.

## D

One of the quadratic factors in part (c) describes motions in which $\xi_1(x,t) = \xi_2(x,t)$, whereas the other describes motions in which $\xi_1(x,t) = -\xi_2(x,t)$. Show that this is true by assuming first that $\xi_1 = \xi_2$ and then that $\xi_1 = -\xi_2$ in parts (a) and (b).

## E

Now suppose that the rollers at $x = 0$ are given the sinusoidal excitation $\xi_1(0,t) = \text{Re}\hat{\xi} e^{j\omega t} = -\xi_2(0,t)$, where $\hat{\xi}$ is the same real constant for each excitation. Also, $0 = \frac{\partial \xi_1}{\partial x} = \frac{\partial \xi_2}{\partial x}$ at $x = 0$. Find $\xi_1(x,t)$ and $\xi_2(x,t)$.

## F

What voltage $V_0$ is required to make the waves excited in part (e) amplify?

### G

Sketch the spatial dependence of $\xi_1$ and $\xi_2$ at an instant in time with $V_0 = 0$ and with $V_0$ large enough to produce amplifying waves.

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## Problem 11.5 (W&M Prob 10.21)

A perfectly conducting membrane with the tension $S$ and mass per unit area $\sigma_m$ is ejected from a nozzle along the $x$-axis with velocity $U$. Gravity acts as shown in Fig. 10P.21.

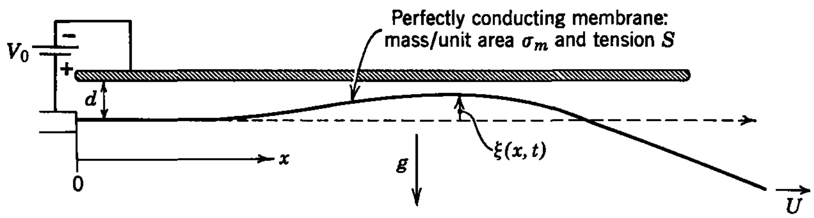


**Fig. 10P.21**

Figure 5: A perfectly conducting membrane moving at velocity $U$.

A planar electrode above the membrane has the constant potential $V_0$ relative to the membrane.

### A

What value of $V_0$ is required to make the membrane assume an equilibrium parallel to the electrode?

### B

Now, under the conditions in (a), the membrane is excited at the frequency $\omega_d$; what is the lowest frequency of excitation that will not lead to spatially growing deflections? Assume that $U > \sqrt{S/\sigma_m}$.

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