6.641 Electromagnetic Fields, Forces, and Motion
Spring 2005

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Problem Set 10 - Questions

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Problem 10.1 (W&M Prob 9.1)

A long thin steel cable of unstressed length l is hanging from a fixed support, as illustrated in Fig. 9P.1. Assume that the origin of coordinates is at the support and that x measures positive as shown. Assume that the steel cable has the following constants.

Cross-sectional area	$A = 10^{-4} m^2$
Young's modulus	$E = 2.0 \times 10^{11} N/m^2$
Mass density	$\rho = 7.8 \times 10^3 kg/m^3$
Maximum allowable stress	$T_{\max} = 2 \times 10^9 N/m^2$

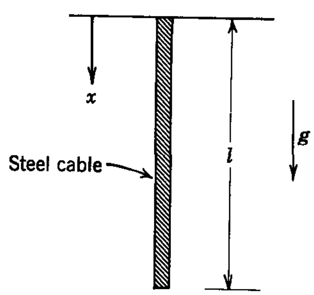

Fig. 9P.1

Figure 1: A long thin steel cable

A

Find the length of cable l for which the maximum stress in the cable just equals the maximum allowable stress.

B

Find the displacement δ and stress T in the cable as functions of x.

C

Find the total elongation of the cable.

Problem 10.2

A thin elastic rod has an initial velocity and stress distribution:

$$v(x, t = 0) = \begin{cases} v_m & 0 < x < a \\ 0 & x > a, x < 0 \end{cases}$$

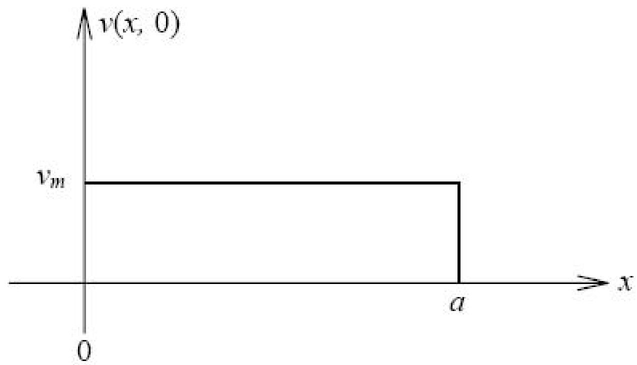

Figure 2: Velocity distribution at time $t = 0$.

$$T(x, t = 0) = 0 \qquad -\infty < x < \infty$$

The rod has Young's modulus E, mass density ρ, and cross-sectional area A.

A

If the rod is of infinite length, $-\infty < x < \infty$, plot the time and space solutions of $v(x, t)$ and $T(x, t)$ in a similar way as shown in Fig. 9.18 of the Woodson/Melcher text.

B

If the rod is of finite length a, $0 < x < a$, with fixed boundaries at $x = 0$ and $x = a$, plot $v(x, t)$ and $T(x, t)$.

Problem 10.3 (W&M Prob 9.6)

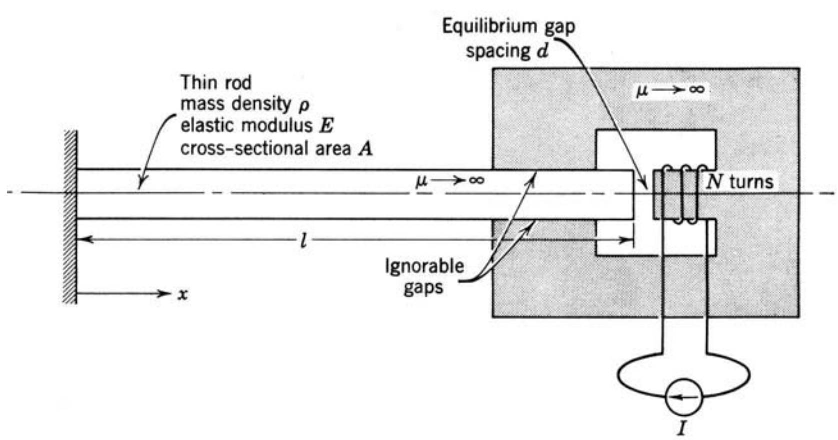

Fig. 9P.6

Figure 3: A thin, circular magnetic rod

A thin, circular magnetic rod is fixed at one end and constrained at the other end by a transducer (Fig 9P.6). In the absence of an excitation, the transducer is simply biased by the constant current source

I. When the rod is in static equilibrium, its length is l and the gap spacing is d. Compute the natural frequencies of the system under the assumption that the magnetization force on the rod acts on the end surface. A graphical representation of the eigenfrequencies is an adequate solution.

Problem 10.4 (W&M Prob 9.10)

A long thin rod is fixed at $x = 0$ and driven at $x = l$, as shown in Fig. 9P.10. The driving transducer consists of a rigid plate with area A attached to the end of the rod, where it undergoes the displacement $\delta(l, t)$ from an equilibrium position exactly between two fixed plates. These fixed plates are biased by potentials V_0 and driven by the voltage $v = \text{Re}(\hat{V} e^{j\omega t})$ as shown. ($|\hat{V}| \ll V_0$)

A

Derive a boundary condition relating $\delta(l, t)$, $(\frac{\partial\delta}{\partial x})(l, t)$ and $v(t)$.

B

Compute the driven deflection of the rod $\delta(x, t)$.

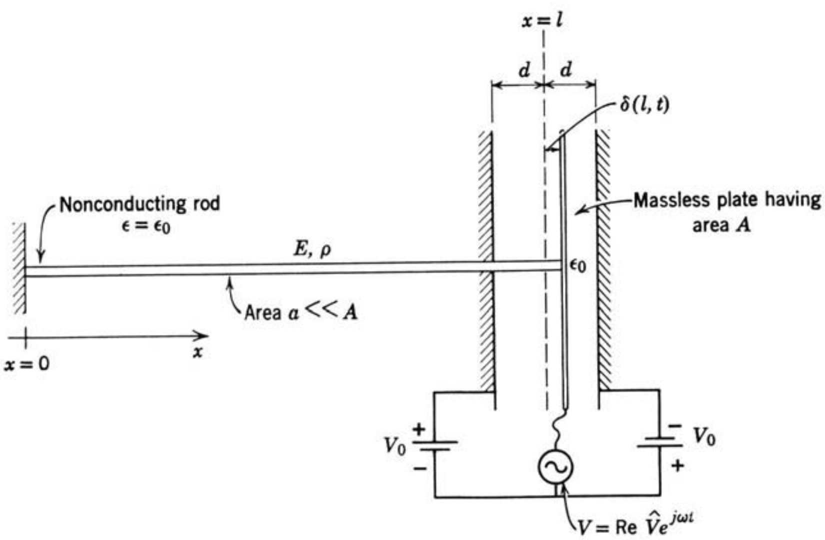

Fig. 9P.10

Figure 4: A long thin rod fixed at $x = 0$ and driven by imposed voltages at $x = l + \delta(l, t)$

Problem 10.5 (Zahn Chap. 8, Prob. 5)

An unusual type of distributed system is formed by series capacitors and shunt inductors.

A

What are the governing partial differential equations relating the voltage and current?

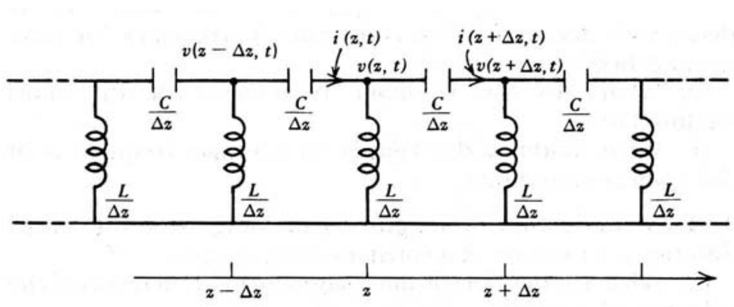

Figure 5: Unusual distributed system

B

What is the dispersion relation between ω and k for signals of the form $e^{j(\omega t-kx)}$?

C

What are the group and phase velocities of the waves? Why are such systems called "backward waves"?

D

A voltage $V_0 \cos \omega t$ is applied at $z=-l$ with the $z=0$ end short circuited. What are the voltage and current distributions along the line?

E

What are the resonant frequencies of the system?

Problem 10.6 (Zahn Chap. 8, Prob. 8)

The dc steady state is reached for a transmission line loaded at $z=l$ with a resistor R_L and excited at $z=0$ by a dc voltage V_0 applied through a source resistor R_s. The voltage source is suddenly set to zero at $t=0$.

A

What is the initial voltage and current along the line?

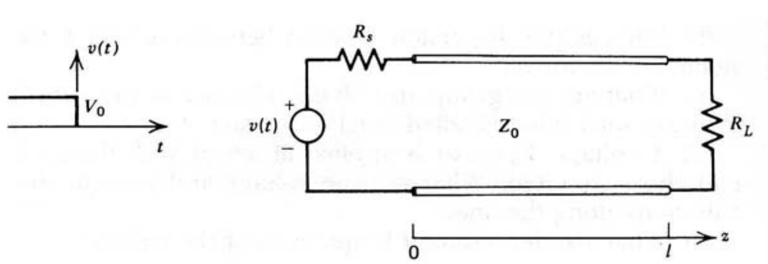

Figure 6: Transmission line system.

B

Find the voltage at the $z = l$ end as a function of time. (Hint: Use difference equations.)