6.641 Electromagnetic Fields, Forces, and Motion
Spring 2005

6.641 — **Electromagnetic Fields, Forces, and Motion** Spring 2005

Problem Set 5 - Solutions

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# Problem 5.1

$$m\frac{d\overrightarrow{v}}{dt} = (q\overrightarrow{v}) \times (\mu_0\overrightarrow{H}); \overrightarrow{H} = H_0\hat{i}_z$$

$$m\frac{d\overrightarrow{v}}{dt} = q\mu_0\begin{vmatrix} \hat{i}_x & \hat{i}_y & \hat{i}_z \\ v_x & v_y & v_z \\ 0 & 0 & H_0 \end{vmatrix}$$

$$\frac{d\overrightarrow{v}}{dt} = \frac{q\mu_0}{m}(v_y H_0\hat{i}_x - v_x H_0\hat{i}_y)$$

**(i)**

$$\frac{dv_x}{dt} = \frac{q\mu_0 v_y}{m}H_0$$

**(ii)**

$$\frac{dv_y}{dt} = \frac{-qmu_0 v_x}{m}H_0$$

$$(i) \Rightarrow \frac{d^2v_x}{dt^2} = \frac{q\mu_0 H_0}{m}\frac{dv_y}{dt}$$

Substitute this into $(ii)$:

$$\frac{m}{q\mu_0 H_0}\frac{d^2v_x}{dt^2} = -\frac{q\mu_0 v_x}{m}H_0$$

$$\frac{d^2v_x}{dt^2} = -\frac{q^2\mu_0^2H_0^2}{m^2}v_x$$

$$v_x = A\cos\left(\frac{q\mu_0 H_0}{m}t\right) + B\sin\left(\frac{q\mu_0 H_0}{m}t\right)$$

$$v_y = C\cos\left(\frac{q\mu_0 H_0}{m}t\right) + D\sin\left(\frac{q\mu_0 H_0}{m}t\right)$$

$$v_x(t=0) = v_{x_0} = A$$

$$v_y(t=0) = v_{y_0} = C$$

Need two more initial conditions:

I. Acceleration in $x$ direction at $t = 0$

$$\hat{i}_x \left.\frac{dv_x}{dt}\right|_{t=0} = \frac{(qv_{y_0}\hat{i}_y) \times (H_0\hat{i}_z)\mu_0}{m}$$

$$B\frac{\cancel{q\mu_0 H_0}}{\cancel{m}} = \frac{\cancel{q\mu_0 H_0}}{\cancel{m}}v_{y_0}$$

$$B = v_{y_0}$$

II. Acceleration in $y$ direction at $t = 0$

$$\hat{i}_y \left.\frac{dv_y}{dt}\right|_{t=0} = \frac{(qv_{x_0}\hat{i}_x) \times (\mu_0 H_0 \hat{i}_z)}{m}$$

$$D = -v_{x_0}$$

$$v_x(t) = v_{x_0}\cos\left(\frac{q\mu_0 H_0}{m}t\right) + v_{y_0}\sin\left(\frac{q\mu_0 H_0}{m}t\right)$$

$$v_y(t) = v_{y_0}\cos\left(\frac{q\mu_0 H_0}{m}t\right) - v_{x_0}\sin\left(\frac{q\mu_0 H_0}{m}t\right)$$

$$v_z(t) = v_{z_0}$$

Note: $\sqrt{v_x^2 + v_y^2 + v_z^2}$ = constant in time (for this case) (easy to check and verify)

$$= \sqrt{v_{x_0}^2 + v_{y_0}^2 + v_{z_0}^2}$$

$$\Rightarrow v_{xy} = \sqrt{v_{x_0}^2 + v_{y_0}^2} = \text{velocity on } xy \text{ plane}$$

From 8.01, centripetal acceleration, $a$, is

$$a = \frac{v_{xy}^2}{r}$$

Where $r$ is radius of circle. So:

$$\underbrace{|m\frac{v_{xy}^2}{r}|}_{F=ma} = \underbrace{|q\overrightarrow{v} \times \mu_0 \overrightarrow{H}|}_{\text{Force due to } \overrightarrow{B} \text{ field}}$$

$$m\frac{v_{xy}^{\not 2}}{r} = q\not{v_{xy}}\mu_0 H_0$$

$$r = \frac{mv_{xy}}{q\mu_0 H_0} = \frac{m\sqrt{v_{x_0}^2 + v_{y_0}^2}}{q\mu_0 H_0}$$

## A

### i

$$\overrightarrow{f} = 0 = q\overrightarrow{E} + q\overrightarrow{v} \times \mu_0 \overrightarrow{H}$$

$$0 = q\overrightarrow{E} + q(v_0\hat{i}_y) \times (\mu_0 H_0 \hat{i}_z)$$

$$\overrightarrow{E} = -v_0\mu_0 H_0 \hat{i}_x$$

$$V = -v_0 \mu_0 H_0 s$$

### ii

$$d = 2r = \frac{2mv_0}{q\mu_0 H_0}; v_0 = \frac{V}{-\mu_0 H_0 s}$$

$$d = \frac{2m|v|}{qB_0^2 s} = 0.50\text{cm for } 12\text{Mg}^{24}$$

$$0.52\text{cm for } 12\text{Mg}^{25}$$

$$0.54\text{cm for } 12\text{Mg}^{26}$$

## B

**i**

$$m\frac{d\overrightarrow{v}}{dt} = q\overrightarrow{E} + q\overrightarrow{v} \times (\mu_0 \overrightarrow{H}) = -e\left(-\frac{V_0}{s}\hat{i}_x + \overrightarrow{v} \times (\mu_0 H_0 \hat{i}_z)\right)$$

$$\overrightarrow{v} \times (\mu_0 H_0 \hat{i}_z) = v_y H_0 \mu_0 \hat{i}_x - v_x H_0 \mu_0 \hat{i}_y$$

$$m\frac{dv_x}{dt} = \frac{V_0 e}{s} - e\mu_0 H_0 v_y$$

$$m\frac{dv_y}{dt} = e\mu_0 H_0 v_x$$

$$\rightarrow m\frac{d^2 v_y}{dt^2} = \frac{e\mu_0 H_0}{m}\left(\frac{eV_0}{s} - e\mu_0 H_0 v_y\right)$$

$$\frac{d^2 v_y}{dt^2} + \left(\frac{e\mu_0 H_0}{m}\right)^2 v_y = \frac{e^2 \mu_0 H_0}{m^2 s} V_0 \qquad (1)$$

$$m\frac{d^2 v_x}{dt^2} = -\frac{e^2 \mu_0^2 H_0^2}{m} v_x$$

$$\frac{d^2 v_x}{dt^2} + \left(\frac{e\mu_0 H_0}{m}\right)^2 v_x = 0 \qquad (2)$$

Solution to equations 1 and 2 (homogenous + particular):

$$v_y = \frac{V_0}{\mu_0 H_0 s}\left(1 + c_1 \sin\left(\frac{e\mu_0 H_0}{m}t\right) + c_2 \cos\left(\frac{e\mu_0 H_0}{m}t\right)\right)$$

$$v_x = \frac{V_0}{\mu_0 H_0 s}\left(c_1 \cos\left(\frac{e\mu_0 H_0}{m}t\right) - c_2 \sin\left(\frac{e\mu_0 H_0}{m}t\right)\right)$$

($v_x$ from $e\mu_0 H_0 v_x = m\frac{dv_y}{dt}$)

$$v_x(t=0) = v_y(t=0) = 0 \Rightarrow c_1 = 0, c_2 = -1$$

$$\overrightarrow{v}(t) = \frac{V_0}{\mu_0 H_0 s}\left(\sin\left[\frac{e\mu_0 H_0}{m}t\right]\hat{i}_x + \left\{1 - \cos\left[\frac{e\mu_0 H_0}{m}t\right]\right\}\hat{i}_y\right)$$

$$\int \overrightarrow{v}(t)dt = \overrightarrow{d}(t) = \frac{mV_0}{\mu_0^2 H_0^2 se}\left(1 - \cos\left[\frac{e\mu_0 H_0}{m}t\right]\right)\hat{i}_x + \left(\frac{V_0}{\mu_0 H_0 s}t - \frac{mV_0}{es\mu_0^2 H_0^2}\sin\left[\frac{e\mu_0 H_0}{m}t\right]\right)\hat{i}_y$$

**ii**

$$|d_x(t)|_{\max} < s \Rightarrow \frac{2mV_0}{\mu_0^2 H_0^2 se} < s$$

$$H_0 > \sqrt{\frac{2mV_0}{\mu_0^2 s^2 e}}$$

# Problem 5.2

## A

$$\oint_C \overrightarrow{H} \cdot \overrightarrow{dl} = \int_S \overrightarrow{J} \cdot \overrightarrow{dA}$$

For contour through space to left of block

$$H_y^L S = NI = NI_0 \cos \omega t$$

$$H_y^L = \frac{NI_0}{s} \sin \omega t$$

For contour through space to right of block

$$H_y^R s = NI_0 \cos \omega t + \cancelto{0}{\int_{\text{Block}} J_z da} \quad \text{(all current in } +z \text{ direction must return in } -z \text{ direction)}$$

$$H_y^R = \frac{NI_0}{s} \sin \omega t$$

## B

$$\nabla \times \overrightarrow{H} = \overrightarrow{J} \tag{3}$$

For <u>Block A</u>,l $\overrightarrow{J} = \sigma \overrightarrow{E}$.

$$\nabla \times \overrightarrow{H} = \sigma \overrightarrow{E}$$

For block B

$$\frac{\partial \overrightarrow{J}}{\partial t} = \omega_p^2 \varepsilon \overrightarrow{E} \tag{4}$$

$$\frac{\partial (\nabla \times \overrightarrow{H})}{\partial t} = \frac{\partial}{\partial t}(\overrightarrow{J})$$

$$\nabla \times \frac{\partial \overrightarrow{H}}{\partial t} = \frac{\partial \overrightarrow{J}}{\partial t} \tag{5}$$

4→5:

$$\nabla \times \frac{\partial \overrightarrow{H}}{\partial t} = \omega_p^2 \varepsilon \overrightarrow{E}$$

## C

Block A

$$\nabla \times \overrightarrow{H} = \sigma \overrightarrow{E}$$

$$\nabla \times \nabla \times \overrightarrow{H} = \nabla \times \sigma \overrightarrow{E}$$

Assume uniform $\sigma$, and use $\nabla \times \nabla \times \overrightarrow{A} = \nabla(\nabla \cdot \overrightarrow{A}) - \nabla^2 \overrightarrow{A}$.

$$\nabla(\nabla \cdot \overrightarrow{H}) - \nabla^2 \overrightarrow{H} = \sigma(\nabla \times \overrightarrow{E})$$

Faraday's Law

$$\nabla \times \vec{E} = -\frac{\partial \mu \vec{H}}{\partial t}$$

Flux continuity: $\nabla \cdot \mu \vec{H} = 0$; for uniform $\mu \Rightarrow \nabla \cdot \vec{H} = 0$.

$$\nabla^2 \vec{H} = \sigma \mu \frac{\partial \vec{H}}{\partial t}$$

Block B

$$\nabla \times \frac{\partial \vec{H}}{\partial t} = \omega_p^2 \varepsilon \vec{E}$$

$$\nabla \times \nabla \times \frac{\partial \vec{H}}{\partial t} = \nabla \times \omega_p^2 \varepsilon \vec{E}$$

Using $\nabla \times \nabla \times \vec{A} = \nabla(\nabla \cdot \vec{A}) - \nabla^2 \vec{A}$ and assuming uniform properties

$$\nabla(\nabla \cdot \frac{\partial \vec{H}}{\partial t}) - \nabla^2 \frac{\partial \vec{H}}{\partial t} = \omega_p^2 \varepsilon (\nabla \times \vec{E})$$

Flux continuity for uniform $\mu \rightarrow \nabla \cdot \vec{H} = 0$ and using Faraday's Law

$$\nabla^2 \frac{\partial \vec{H}}{\partial t} = \omega_p^2 \varepsilon \mu \frac{\partial \vec{H}}{\partial t}$$

Integrate and assume that integration constant is zero

$$\nabla^2 \vec{H} = \omega_p^2 \varepsilon \mu \vec{H} = \frac{\omega_p^2}{c^2} \vec{H}$$

$$c = \frac{1}{\sqrt{\varepsilon \mu}} = \text{ speed of light in material}$$

## D

Assume $\vec{H} = \mathrm{Im}\left\{ \hat{H}_y(x) e^{j\omega t} \hat{i}_y \right\}$

$$\nabla^2 \vec{H} = \sigma \mu \frac{\partial \vec{H}}{\partial t}$$

for Block A.

$$\frac{\partial^2}{\partial x^2} \hat{H}_y(x) e^{j\omega t} = \sigma \mu j \omega \hat{H}_y(x) e^{j\omega t}$$

$$\frac{\partial^2}{\partial x^2} \hat{H}_y(x) = \sigma \mu j \omega \hat{H}_y(x)$$

Assume $\hat{H}_y(x) = \hat{H}_0 e^{jkx}$.

$$-k^2 \cancel{\hat{H}_0 e^{jkx}} = \sigma \mu \cancel{\hat{H}_0 e^{jkx}} j\omega$$

$$-k^2 = \sigma\mu j\omega, k = \pm\sqrt{-\omega\mu j\sigma}$$
$$k = \pm\sqrt{\frac{\sigma\mu\omega}{2}}(1-j)$$
$$\alpha \equiv \sqrt{\frac{\sigma\mu\omega}{2}}$$
$$\overrightarrow{H}_y = \text{Im}\left\{\hat{H}_0^1 e^{\alpha x} e^{j\alpha x} e^{j\omega t} + H_0^2 e^{-\alpha x} e^{-j\alpha x} e^{j\omega t}\right\}$$

Apply B.C. to complex $H$. At $x = 0$, $K = 0 \Rightarrow H_y(x = 0, t) = \frac{NI_0}{s}\sin\omega t$.

$$= \text{Im}\left\{\frac{NI_0}{s} e^{j\omega t}\right\}$$
$$\hat{H}_0^1 e^0 e^0 e^{j\omega t} + \hat{H}_0^2 e^0 e^{j\omega t}$$
$$= \frac{NI_0}{s} e^{j\omega t}$$
$$\hat{H}_0^1 + \hat{H}_0^2 = \frac{NI_0}{s}$$

At $x = d$, $K = 0 \Rightarrow H_y(x = d, t) = \frac{NI_a}{s}\sin\omega t$

$$\hat{H}_0^1 e^{\alpha d} e^{j\alpha d} e^{j\omega t} + \hat{H}_0^2 e^{-\alpha d} e^{-j\alpha d} e^{j\omega t} = \frac{NI_0}{s} e^{j\omega t}$$
$$\Rightarrow \hat{H}_0^1 e^{\alpha d} e^{j\alpha d} + \hat{H}_0^2 e^{-\alpha d} e^{-j\alpha d} = \frac{NI_0}{s}$$

From B.C. at $x = 0$

$$\hat{H}_0^1 = \frac{NI_0}{s} - \hat{H}_0^2$$
$$\frac{NI_0}{s} e^{\alpha d} e^{j\alpha d} - H_0^2\left[e^{\alpha d} e^{j\alpha d} - e^{-\alpha d} e^{-j\alpha d}\right] = \frac{NI_0}{s}$$
$$\hat{H}_0^2 = \frac{\frac{NI_0}{s}\left(1 - e^{\alpha d} e^{j\alpha d}\right)}{\left(e^{-\alpha d} e^{-j\alpha d} - e^{\alpha d} e^{j\alpha d}\right)}$$
$$\hat{H}_0^1 = \frac{NI_0}{s} - \frac{NI_0}{s}\left[\frac{1 - e^{\alpha d} e^{j\alpha d}}{e^{-\alpha d} e^{-j\alpha d} - e^{\alpha d} e^{j\alpha d}}\right]$$
$$= \frac{NI_0}{s}\left[\frac{e^{-\alpha d} e^{-j\alpha d} - 1}{e^{-\alpha d} e^{-j\alpha d} - e^{\alpha d} e^{j\alpha d}}\right]$$
$$\overrightarrow{H} = \text{Re}\left\{\hat{H}_0^1 e^{\alpha x} e^{j\alpha x} e^{j\omega t} + \hat{H}_0^2 e^{-\alpha x} e^{-j\alpha x} e^{j\omega t}\right\}\hat{i}_y$$

Or

$$\overrightarrow{H} = \text{Im}\left[\frac{NI_0}{s}\frac{\cosh\gamma(x - \frac{d}{2})}{\cosh\frac{xd}{2}} e^{j\omega t}\right]\hat{i}_y$$

with $\gamma = \frac{1+j}{\delta}, \delta = \sqrt{\frac{2}{\omega\mu\sigma}}$, $\alpha = \sqrt{\frac{\sigma\mu\omega}{2}} = \frac{1}{\delta}$.

For Block B: Assume $H_y(x, t) = \text{Im}\left\{\hat{H}_0 e^{jkx} e^{j\omega t}\right\}$.

$$\nabla^2\overrightarrow{H} = \omega_p^2\varepsilon\mu\overrightarrow{H}$$

$$\frac{\partial^2}{\partial x^2}\hat{H}_0 e^{jkx}e^{j\omega t} = \omega_p^2\varepsilon\mu\hat{H}_0 e^{jkx}e^{j\omega t}$$
$$-k^2 = \omega_p^2\varepsilon\mu$$
$$k = \pm\sqrt{\omega_p^2\varepsilon\mu} = \pm\omega_p\sqrt{\varepsilon\mu}j$$
$$\vec{H}_y(x) = \text{Im}\left\{\hat{H}_0^1 e^{\alpha x}e^{j\omega t} + \hat{H}_0^2 e^{-\alpha x}e^{j\omega t}\right\}$$
$$\alpha = \omega_p\sqrt{\varepsilon\mu}$$

Apply B.C. at $x = 0$, $K = 0 \Rightarrow H_y^{0^+} = \frac{I_0 N}{s}\sin\omega t$.

$$\hat{H}_0^1 e^0 e^{j\omega t} + \hat{H}_0^2 e^0 e^{j\omega t} = \frac{I_0 N}{s}e^{j\omega t}$$
$$\rightarrow \hat{H}_0{}^1 + \hat{H}_0{}^2 = \frac{NI_0}{s}$$

Apply B.C. at $x = d$, $K = 0 \Rightarrow H_y^d = \frac{I_0 N}{s}\sin\omega t$.

$$\hat{H}_y^1 e^{\alpha d}e^{j\omega t} + \hat{H}_0^2 e^{-\alpha d}e^{j\omega t} = \frac{I_0 N}{s}e^{j\omega t}$$
$$\hat{H}_0^1 e^{\alpha d} + \hat{H}_0^2 e^{-\alpha d} = \frac{NI_0}{s}$$
$$\left[\frac{NI_0}{s} - \hat{H}_0^2\right]e^{\alpha d} + \hat{H}_0{}^2 e^{\alpha d} - \frac{NI_0}{s}$$
$$\hat{H}_0^2\left[e^{-\alpha d} - e^{\alpha d}\right] = \frac{NI_0}{s}\left[1 - e^{\alpha d}\right]$$
$$\hat{H}_0^2 = \frac{NI_0}{s}\left[\frac{1 - e^{\alpha d}}{e^{-\alpha d} - e^{\alpha d}}\right]$$
$$\hat{H}_0^1 = \frac{NI_0}{s}\left[\frac{e^{-\alpha d} - 1}{e^{-\alpha d} - e^{\alpha d}}\right]$$
$$\vec{H}_y(x) = \text{Im}\left\{\hat{H}_0^1 e^{\alpha x}e^{j\omega t} + \hat{H}_0^2 e^{-\alpha x}e^{j\omega t}\right\}$$
$$\alpha = \omega_p\sqrt{\varepsilon\mu}$$

Or

$$H_0^2 = \frac{NI_0}{s}\left[\frac{1 - e^{\alpha d}}{e^{-\alpha d} - e^{\alpha d}}\right]$$
$$H_0^1 = \frac{NI_0}{s}\left[\frac{e^{-\alpha d} - 1}{e^{-\alpha d} - e^{\alpha d}}\right]$$
$$H_y(x,t) = \left[H_0^1 e^{\alpha x} + H_0^2 e^{-\alpha x}\right]\sin\omega t$$
$$H_y(x,t) = \frac{NI_0}{s}\frac{\cosh\alpha(x - \frac{d}{2})}{\cosh\frac{\alpha d}{2}}\sin\omega t, \qquad \text{with } \alpha = \frac{\omega_p}{c} = \omega_p\sqrt{\varepsilon\mu}$$

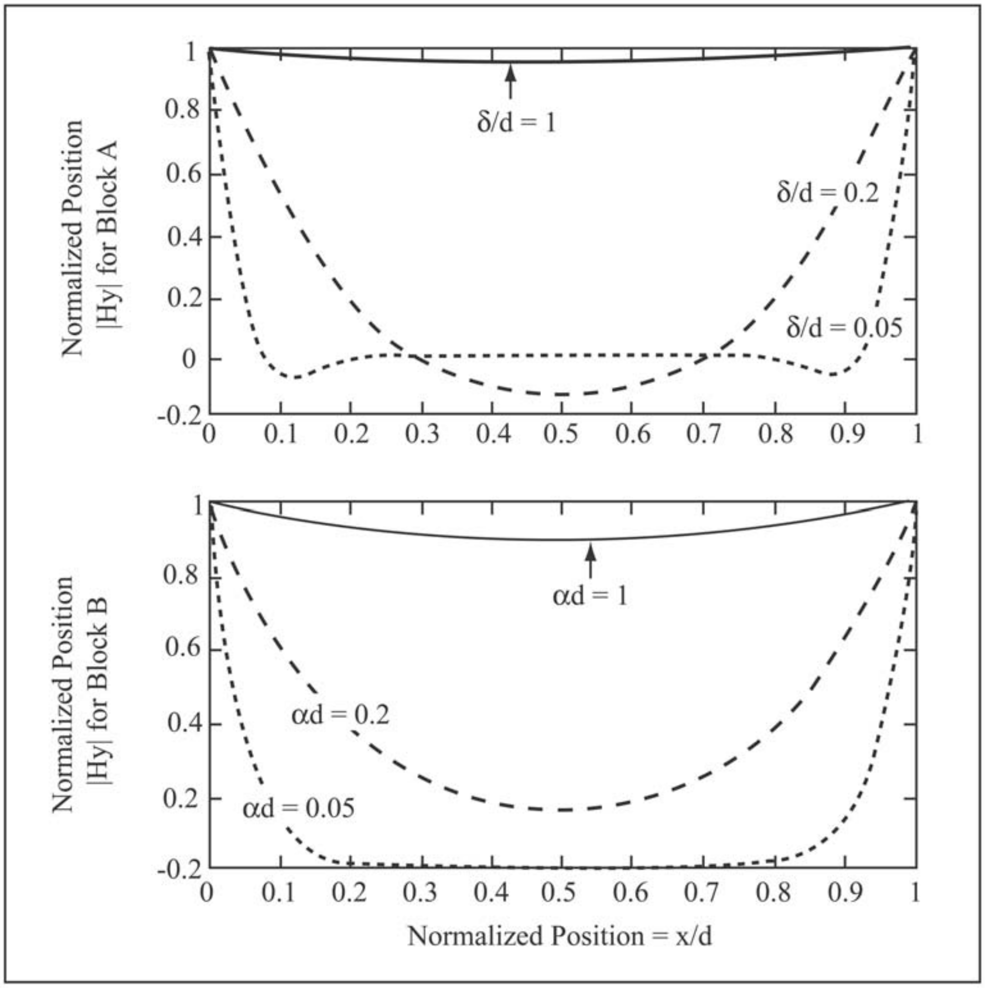


Figure 1: $|H_y|$ for blocks A and B for Problem 5.2 (Image by MIT OpenCourseWare.)

## E

For Block A

$$\nabla \times \overrightarrow{H} = \overrightarrow{J}$$

Since $\overrightarrow{H} = \overrightarrow{H}_y(x)$

$$\overrightarrow{J} = \hat{z}\frac{\partial H_y(x)}{\partial x}$$

$$\overrightarrow{J} = \mathrm{Im}\left\{\hat{H}_0^1(\alpha + j\alpha)e^{\alpha x}e^{j\alpha x}e^{j\omega t} + \hat{H}_0^2(-\alpha - j\alpha)e^{-\alpha x}e^{-j\alpha}e^{j\omega t}\right\}\hat{z} = \mathrm{Im}\left[\frac{NI_0}{s}\frac{\gamma \sinh\gamma(x - \frac{d}{2})}{\cosh(\frac{\gamma d}{2})}e^{j\omega t}\right]\hat{i}_z$$

For Block B

$$\overrightarrow{J} = \hat{z}\frac{\partial H_y(x)}{\partial y}$$

$$\overrightarrow{J} = \text{Im}\left\{\hat{H}_0^1 \alpha e^{\alpha x} e^{j\omega t} - \hat{H}_0^2 \alpha e^{-\alpha x} e^{j\omega t}\right\} \hat{z}$$

or

$$\overrightarrow{J}(x,t) = \left[H_0^1 \alpha e^{\alpha x} - H_0^2 \alpha e^{-\alpha x}\right] \sin \omega t \hat{z}$$

$$= \frac{NI_0}{s} \frac{k \sinh k(x - \frac{d}{2})}{\cosh \frac{kd}{2}} \sin \omega t \hat{i}_z$$

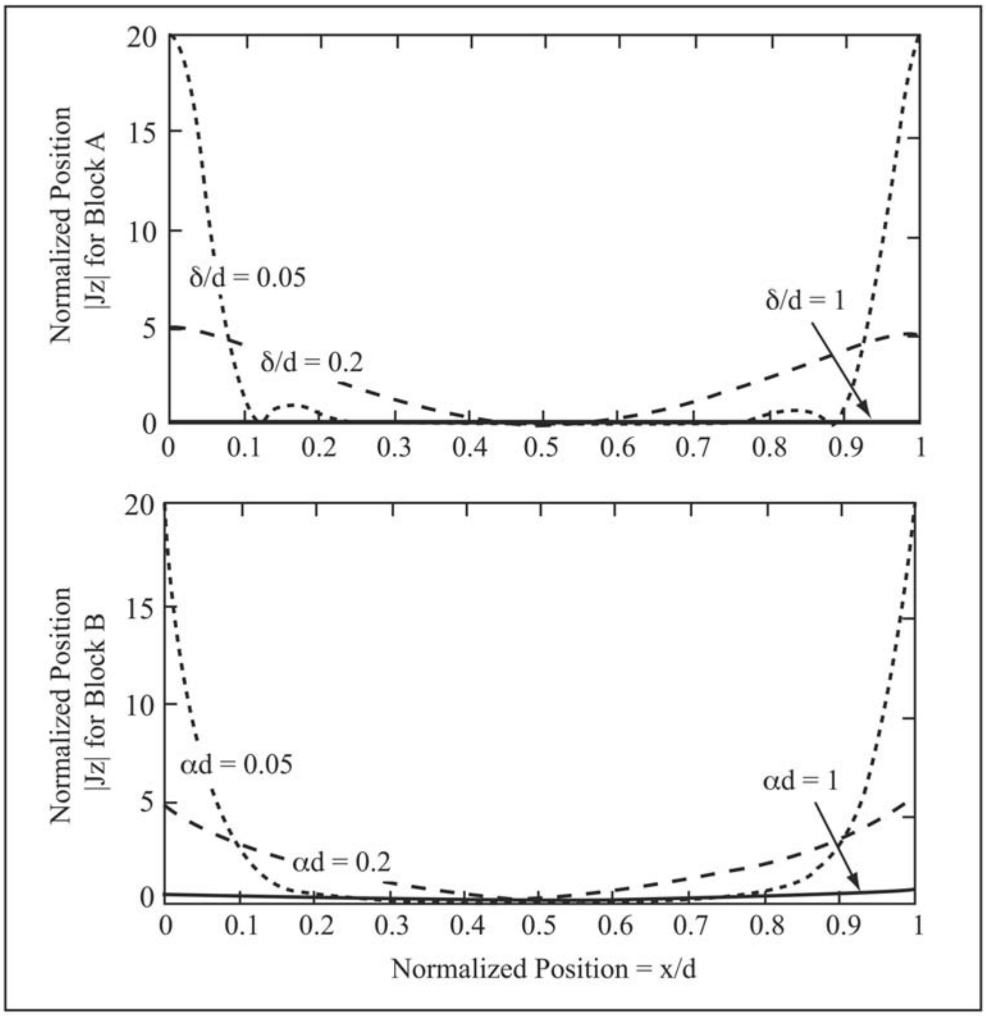


Figure 2: $|J_z|$ for blocks A and B for Problem 5.2 (Image by MIT OpenCourseWare.)

# Problem 5.3

## A

From the Boundary condition $\overline{i}_x \cdot \left[\varepsilon_0 \bar{E}(x = 0^+) - \varepsilon_0 \overline{E}(x = 0^-)\right] = \sigma_s$, we know

$$E_x(x = 0^+) - E_x(x = 0^-) = \frac{\sigma_s}{\varepsilon_0}$$

and from the symmetry, we know $E_x(x=0^+) = -E_x(x=0^-)$. So $E_x(x=0^+) = -E_x(x=0^-) = \frac{\sigma_s}{2\varepsilon_0} = \frac{\sigma_0\cos(ay)}{2\varepsilon_0}$. We can build the boundary condition for the scalar potential:

$$BC1 : \Phi(x \to \infty) = 0, \Phi(x \to -\infty) = 0$$

$$BC2 : -\frac{\partial\Phi(x,y)}{\partial x}(x=0^+) = +\frac{\partial\Phi(x,y)}{\partial x}(x=0^-) = \frac{\sigma_0\cos(ay)}{2\varepsilon_0}$$

Here we assume the general solution for the scalar potential is given: $\Phi(x,y) = AX(x)\Psi(y)$. As BC1 implied, $X(x) = e^{-ax}, x > 0$; and $e^{ax}, x < 0$. With BC2, we find the scalar potential as:

$$\Phi(x,y) = \frac{\sigma_0 e^{-ax}\cos(ay)}{2a\varepsilon_0}, x > 0$$

$$\Phi(x,y) = \frac{\sigma_0 e^{ax}\cos(ay)}{2a\varepsilon_0}, x < 0$$

With $\bar{E} = -\nabla\Phi = -\frac{\partial\Phi}{\partial x}\bar{i}_x - \frac{\partial\Phi}{\partial y}\bar{i}_y$.

$$\bar{E} = \frac{\sigma_0 e^{-ax}}{2\varepsilon_0}(\cos(ay)\bar{i}_x + \sin(ay)\bar{i}_y), x > 0$$

$$\bar{E} = -\frac{\sigma_0 e^{ax}}{2\varepsilon_0}(\cos(ay)\bar{i}_x - \sin(ay)\bar{i}_y), x < 0$$

### B

For the electric field line, we have $\frac{dy}{dx} = \frac{E_y}{E_x}$. At $x > 0$,

$$dx = \frac{\cos(ay)}{\sin(ay)}dy \Rightarrow e^{-ax}\sin(ay) = \text{ constant}$$

At $x < 0$,

$$dx = -\frac{\cos(ay)}{\sin(ay)}dy \Rightarrow e^{ax}\sin(ay) = \text{ constant}$$

### C

See Figure 3

## Problem 5.4

### A

At region $x > 0$, we have the general solution $\Psi_I = e^{-ax}A\sin(ay)$. At region $x < 0$, we have solution $\Psi_{II} = e^{ax}B\sin(ay)$. We chose exponentials in $x$ for 0 potential at $x = \pm\infty$. We chose sines as $\bar{H} = -\nabla\Psi$ will have to satisfy $\cos(ay)$ and the boundary condition for $x = 0$.

### B

Boundary Condition 1: $\bar{n}\cdot\left[\bar{B}_I - \bar{B}_{II}\right] = 0 \Rightarrow \mu_0 H_{Ix}|_{x=0+} = \mu H_{IIx}|_{x=0-}$.
Boundary Condition 2: $\bar{n}\times\left[\bar{H}_1 - \bar{H}_{II}\right] = K_0\cos(ay)\bar{i}_x$.

$$H_{Iy}|_{x=0+} - H_{IIy}|_{x=0-} = K_0\cos(ay)$$

Here $\bar{H} = -\nabla\Psi, \bar{H}_I = ae^{-ax}A\sin(ay)\bar{i}_x - ae^{-ax}A\cos(ay)\bar{i}_y$

$$\bar{H}_{II} = -ae^{ax}B\sin(ay)\bar{i}_x - ae^{ax}B\cos(ay)\bar{i}_y$$

By BC1: $\mu_0 A = -\mu B$; By BC2: $B - A = \frac{K_0}{a}$. We can solve: $A = -\frac{\mu K_0}{a(\mu_0+\mu)}$ and $B = \frac{\mu_0 K_0}{a(\mu_0+\mu)}$.

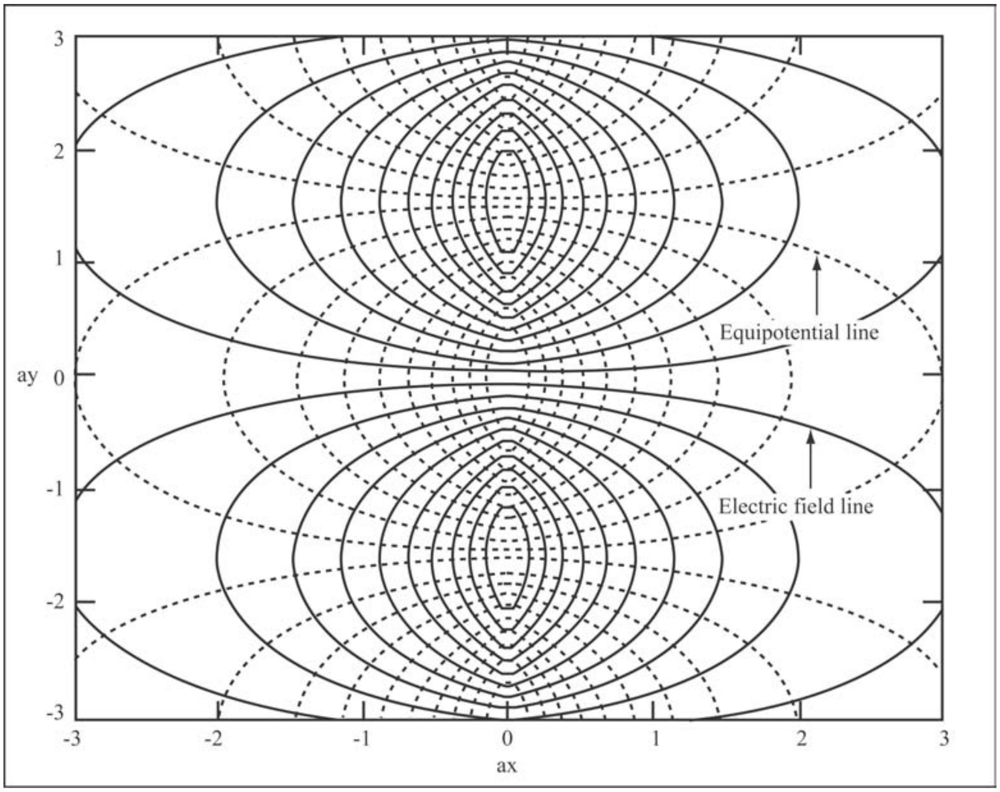


Figure 3: Electric Field Lines and Equipotential Lines for Problem 5.3C (Image by MIT OpenCourseWare.)

## C

The solution for $H$ field:

$$\bar{H}_1 = \frac{\mu K_0}{(\mu_0 + \mu)} \left[ -e^{-ax} \sin(ay) \bar{i}_x + e^{-ax} \cos(ay) \bar{i}_y \right]$$

$$\bar{H}_{II} = \frac{\mu_0 K_0}{(\mu_0 + \mu)} \left[ -e^{ax} \sin(ay) \bar{i}_x - e^{ax} \cos(ay) \bar{i}_y \right]$$