

## Problem Set 6 Solutions

### Problem 1

- (a) The first formant of the configuration is the Helmholtz resonance. The second formant frequency is the lowest natural frequency of the back cavity since  $l_b$  is much larger than  $l_f$ .

$$\text{In this case: } F_2 = \frac{c}{2l_b} = \frac{35400 \frac{cm}{s}}{2 * 13cm} = \boxed{1361.54 \text{ Hz}}$$

- (b) Lowest natural frequency of the front cavity :  $F = \frac{c}{4l_f} = \frac{35400 \frac{cm}{s}}{4 * 1.9cm} = \boxed{4657.9 \text{ Hz}}$

To determine the index of this formant, we list the natural frequencies of the back cavity and see how many are below F.

$$F_3 = \frac{c}{l_b} = \frac{35400 \frac{cm}{s}}{13cm} = 2723.1 \text{ Hz}$$

$$F_4 = \frac{3c}{2l_b} = \frac{3 * 35400 \frac{cm}{s}}{2 * 13cm} = 4084.6 \text{ Hz}$$

$$\boxed{F = F_5 = 4657.9 \text{ Hz}}$$

$$F_6 = \frac{4c}{2l_b} = \frac{4 * 35400 \frac{cm}{s}}{2 * 13cm} = 5446.2 \text{ Hz}$$

- (c) (i) Frequency of the pole is the lowest natural frequency:

$$F_p = \frac{c}{4l_f} = \frac{35400 \frac{cm}{s}}{4 * 1.9cm} = \boxed{4657.9 \text{ Hz}}$$

- (ii) This part is similar to the Problem 2a from problem set 4, recall from problem set 4:

$$\frac{U_o}{P_s} = j \frac{A \sin(kl_s)}{\rho c \cos(kl_f)} \quad (1)$$

The poles of this expression are the natural frequencies of the entire tube, and the zeros are the frequencies at which the impedance looking to the left of the pressure source is infinite.

For low frequencies,  $\sin(kl_s) \approx kl_s$  and  $\cos(kl) \approx 1$ ,

Eq. (1) simplifies to

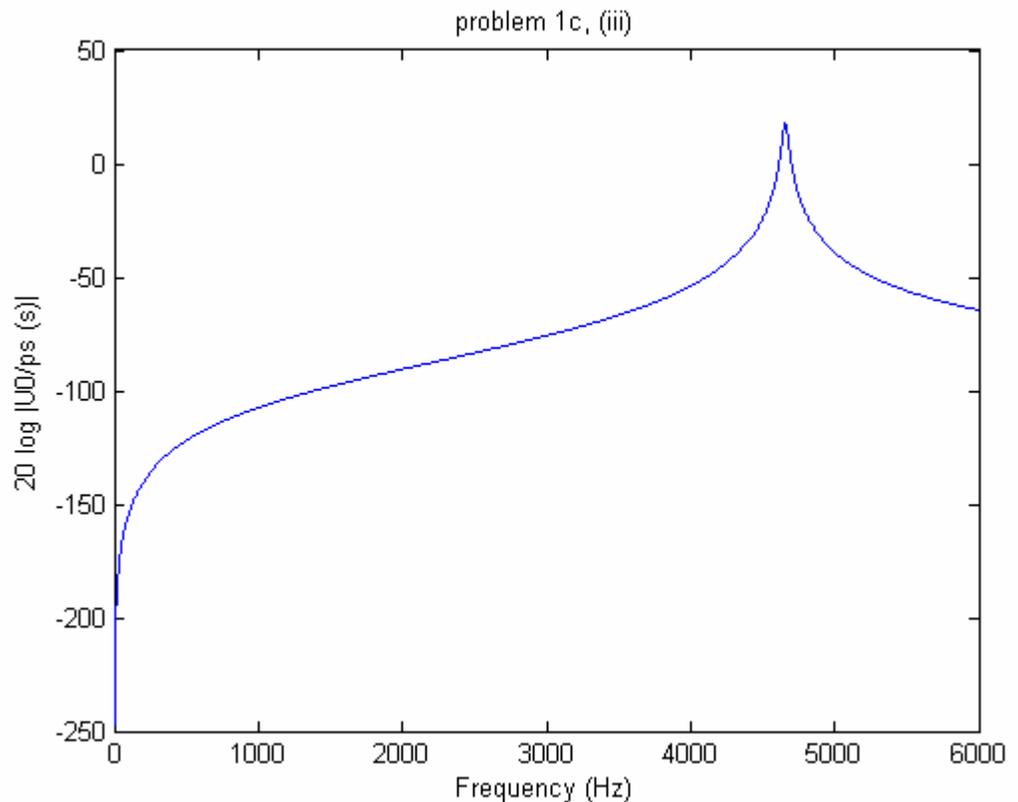
$$\frac{U_o}{P_s} = j \frac{A}{\rho c} \frac{kl_s}{1} = j 2\pi f \frac{Al_s}{\rho c^2} = Ks,$$

$$\text{where } K = \frac{Al_s}{\rho c^2} = \frac{1 * 1}{0.00114 * 35400^2}$$

$$\boxed{K = 7E-7}$$

(iii) Two sets of answers will be accepted for this part.

1. The following plot is obtained if one simply plugged all the values into the expression given in the problem. Please see attached matlab code.



2. The second approach is by dividing the expression into three parts,

$$20\log\left(\frac{U_0}{P_s}\right) = 20\log(K) + 20\log(s) + 20\log(|T(s)|) \quad (2)$$

$20\log(|T(s)|)$  is in Figure 1.3.

$20\log(K) = -123\text{dB}$   $\Rightarrow$  shift Figure 1.3 down by 123 dB.

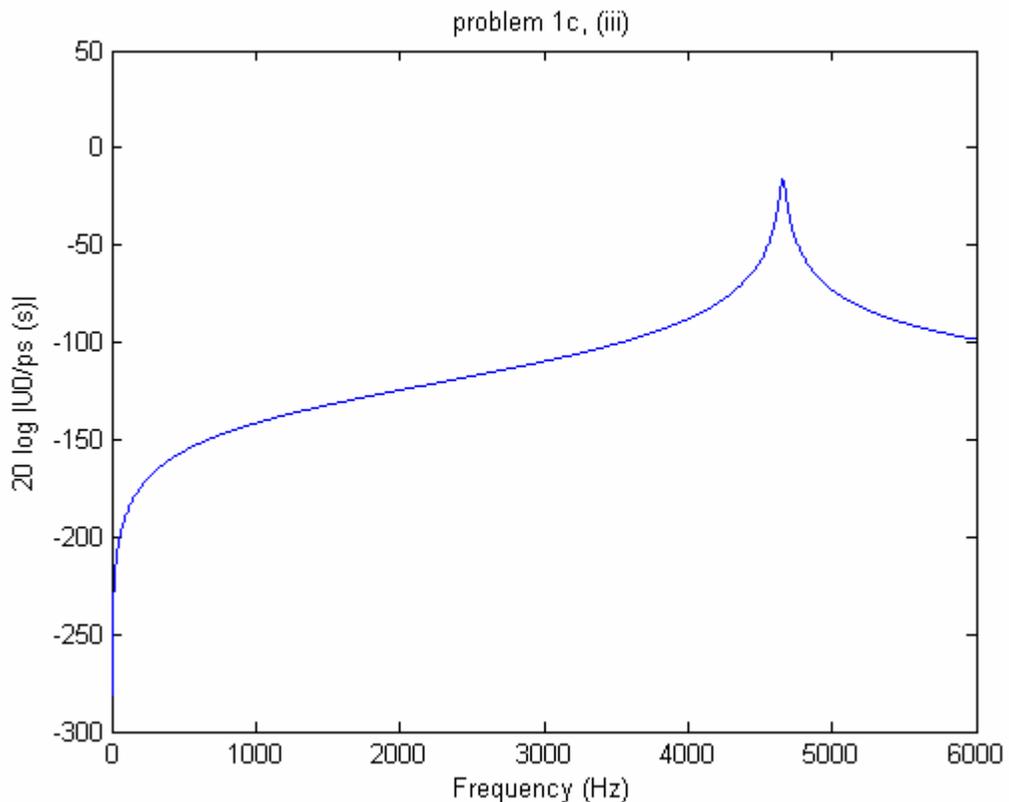
$20\log(s)$ :

Zero at the origin will cause the curve to rise up at a steeper rate near DC.

Also, at the location of the pole = 4657.9 Hz,

$$20\log(s) = 20\log(\text{abs}(2*\pi*j*4657.9)) = 89\text{dB}.$$

$\Rightarrow$  the first two terms of eq.(2) causes Figure 1.3 to shift down by a total of  $123-89= 34$  dB at the pole location.



The reason why these two methods would give different answers is because Figure 1.3 plots the transfer function as a function of frequency ( $f$ ) instead of  $s = 2\pi f$ , so there is a DC offset between the two graphs. Both answers will be accepted as long as the method is correct.

- (iv) If  $l_s = 0.5$  cm instead of 1.0 cm.  $K$  would decrease by a factor of 2, which would shift the magnitude of  $U_0/p_s$  vs. frequency curve down by 6dB.

### Problem 2

- (a) A: There is a decrease in the amount of high frequency component in the waveform, nasalization;

=>point 3

B: Based on the formant frequencies:

=>point 1

C: Tongue position is back (low F2) and low (High F1)

=>point 1

- (b) A total of 10 harmonics fit into 1650 Hz =>  $F_0 = 165$  Hz

- (c) Since F3 does not vary as much as F1 and F2, we can use it to approximate the length of the vocal tract by setting F3 to be the 3<sup>rd</sup> natural frequency of a tube that is open on one end.

$$F_3 = \frac{5c}{4l} = 2900\text{Hz} \quad \Rightarrow \quad l = 15.26 \text{ cm}$$

### Problem 3

1. We were on war memorial way.
2. Mary will run a mile.
3. We will rule no more.

Matlab script for Problem 1c(iii);

```
f = 0:1:6000;
s = j*2*pi*f;
K = 7E-7;

Fn = 4657.9;
Bn = Fn/12;
sigma_n = Fn/12/pi;
s_p = sigma_n + j*2*pi*Fn;
s_p_conj = sigma_n - j*2*pi*Fn;
func = K*s*s_p*s_p_conj./(s - s_p)./(s - s_p_conj);

mag = 20*log(abs(func));
plot(f, mag)
title('problem 1c, (iii)');
xlabel('Frequency (Hz)');
ylabel('20 log |U0/psl')
```