Problem Set 2 Solutions

Problem 1

(a) False
Contraction of the lower fibers of the genioglossus muscle increases the cross-sectional area of the pharynx. (pg 15-16)

(b) True.
The tension on the vocal folds is increased by contracting the cricothyroid muscle. (pg 7-8)

(c) True
The mandible is usually in a higher position for the sound /s/ than for the sound /i/. (try it)

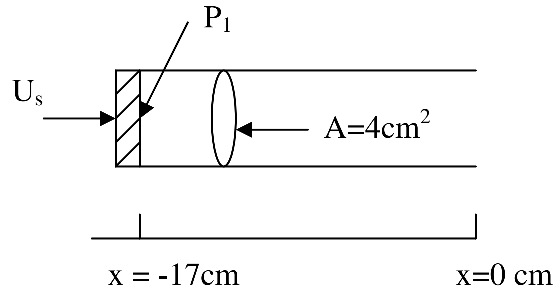
(d) False
On average, the ratio of the length of the oral cavity to the length of the pharyngeal cavity is greater for adult females than for adult males. (pg 24-25)

(e) False
Contraction of the sternohyoid muscle pulls the larynx downwards. (pg 13-14)

(f) False
Downward displacement of the larynx results in a shortening of the vocal folds. (pg13-14)

(g) True
The cross-sectional area of the vocal tract is greater in the pharyngeal region for the vowel /i/ than it is for the vowel /ɑ/. (pg 16-17)

Problem 2



Recall the following relations from lecture:

1-Dimensional Wave Equation: $\dfrac{d^2 p(x)}{dx^2} = -k^2 p(x)$, where $k = \dfrac{\omega}{c}$ **(1)**

Newton's Law: $\dfrac{dp(x)}{dx} = -\dfrac{j\omega\rho}{A} U(x)$ **(2)**

We also have the following boundary conditions:

p(x=0cm) = 0 dyne/cm^2
|U(x=-17cm)| = U_s = 100 cm^3/s

1) Knowing that p(x) is a sinusoid, based on the first boundary condition, we guess a solution for p(x) of the form p(x)=$P_m$sin(αx). We have to figure out what P_m and α are.

2) By plugging our guessed solution into equation (1), we can get α.

$$\frac{d^2 p(x)}{dx^2} = -\alpha^2 P_m \sin(\alpha x)$$

$$-\alpha^2 P_m \sin(\alpha x) = -\left(\frac{\omega}{c}\right)^2 P_m \sin(\alpha x)$$

$$\Rightarrow \alpha = \frac{\omega}{c}$$

we now have p(x) = $P_m \sin(\frac{\omega}{c} x)$

3) Using equation(2), we get

$$U(x) = -\frac{A}{j\omega\rho}\frac{dp(x)}{dx} = -\frac{A}{j\omega\rho}\left(\frac{\omega}{c}\right) P_m \cos\left(\frac{\omega}{c} x\right)$$

4) Applying the second boundary condition, we have

$U_s = |U(x=-17cm)| = 100\ cm^3/s$

$$U_s = \left| \frac{jA}{\rho c} P_m \cos\left(\frac{\omega}{c}(-17cm) \right) \right|$$

$$\Rightarrow P_m = \frac{U_s \rho c}{A \cdot \left| \cos\left(\frac{\omega}{c}(-17cm) \right) \right|}$$

5) Finally, to solve for P_1, where $P_1 = |p(x = -17cm)|$, we plug in all the values, and get

$$P_1 = \left| P_m \sin\left(\frac{\omega}{c}(-17cm) \right) \right| = \frac{U_s \rho c}{A} \tan\left(\frac{\omega}{c}(17cm) \right)$$

$$= \frac{(100cm^3/s)(0.00114g/cm^3)(35400cm/s)}{4cm^2} \tan\left(\frac{2\pi(400Hz)(17cm)}{35400cm/s} \right)$$

$$\Rightarrow \boxed{P_1 = 2649.335\ \text{dynes/cm}^2}$$

Problem 3

a) $F = \frac{c}{4l}, \frac{3c}{4l}, \frac{5c}{4l}, \frac{7c}{4l}$, where c= 354000 cm/s, l = 15cm

$\boxed{F1 = 590\text{ Hz},\quad F2 = 1770\text{ Hz},\quad F3 = 2950\text{ Hz},\quad F4 = 4130\text{ Hz}}$

b) $Z_L + Z_R = 0$

$$-j\frac{\rho c}{A}\cot\left(\frac{2\pi f}{c}l\right) + j2\pi f\frac{\rho l_c}{A_c} = 0 \qquad \text{where} \quad \begin{aligned} A &= 4cm^2 \\ A_c &= 2cm^2 \\ l &= 15cm \\ l_c &= 2cm \end{aligned}$$

Using Matlab to solve for f (refer to the script attached), we get $\boxed{F1 = 469.22\text{ Hz},\ F2 = 1467.3\text{ Hz}}$, which makes sense qualitatively, because a lengthened tube would have lower natural frequencies.

c)

i) $\tan x \cong \frac{1}{\frac{\pi}{2} - x}$ when $x \cong \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}$, etc..

For F1= 590 Hz: $\frac{\rho c}{A} \cdot \frac{1}{\frac{\pi}{2} - \frac{\omega \ell}{c}} \cong -\omega_1 \frac{\rho \ell_c}{A_c}$,

$$=> \omega = \left(\frac{\pi}{2} + \frac{A_c c}{A \varpi_1 l_c} \right) \cdot \frac{c}{l} \quad \text{where} \quad \begin{array}{l} A = 4cm^2 \\ A_c = 0.02cm^2 \\ l = 15cm \\ l_c = 0.3cm \\ F_1 = 590Hz \end{array}$$

$$2\pi F_1' = \left(\frac{\pi}{2} + \frac{A_c c}{A 2\pi F_1 l_c} \right) \cdot \frac{c}{l}$$

=> $\boxed{F_1' = 649.78 \text{ Hz}}$

Similarly for F2 = 1770 Hz:

$$2\pi F_2' = \left(\frac{3\pi}{2} + \frac{A_c c}{A 2\pi F_1 l_c} \right) \cdot \frac{c}{l}$$

=> $\boxed{F_2' = 1789.93 \text{ Hz}}$

We must also check to see if the tangent approximation is valid:

Natural Frequency	ExactTan	Tan. Approximation	Difference
649.78 Hz	-6.23	-6.28	0.05
1789.93 Hz	-18.83	-18.85	0.02

The approximation is valid and it gets better as frequency increases.

ii) From the attached graph, $\boxed{F_1' = 644.35 \text{ Hz and } F_2' = 1789.7 \text{ Hz}}$
This is pretty close to what we get from (i).

iii) See attached MATLAB script. $\boxed{F_1' = 644.35 \text{ Hz and } F_2' = 1789.7 \text{ Hz,}}$

d). Treating the small tube as a Helmholtz resonator since the dimensions are small compared with the wavelength, we know that the natural frequency is

$$F = \frac{c}{2\pi} \cdot \sqrt{\frac{A_c}{l_c A(\Delta l)}} \quad , \text{ where } \quad A = 4cm^2,\; A_c = 0.02cm^2,\; l = 15cm$$

$$\Rightarrow \quad \Delta l = \left(\frac{c}{2\pi F}\right)^2 \cdot \frac{A_c}{A l_c}$$

Setting F = 590Hz, the first natural frequency of the original tube, we get Δl = 1.52 cm

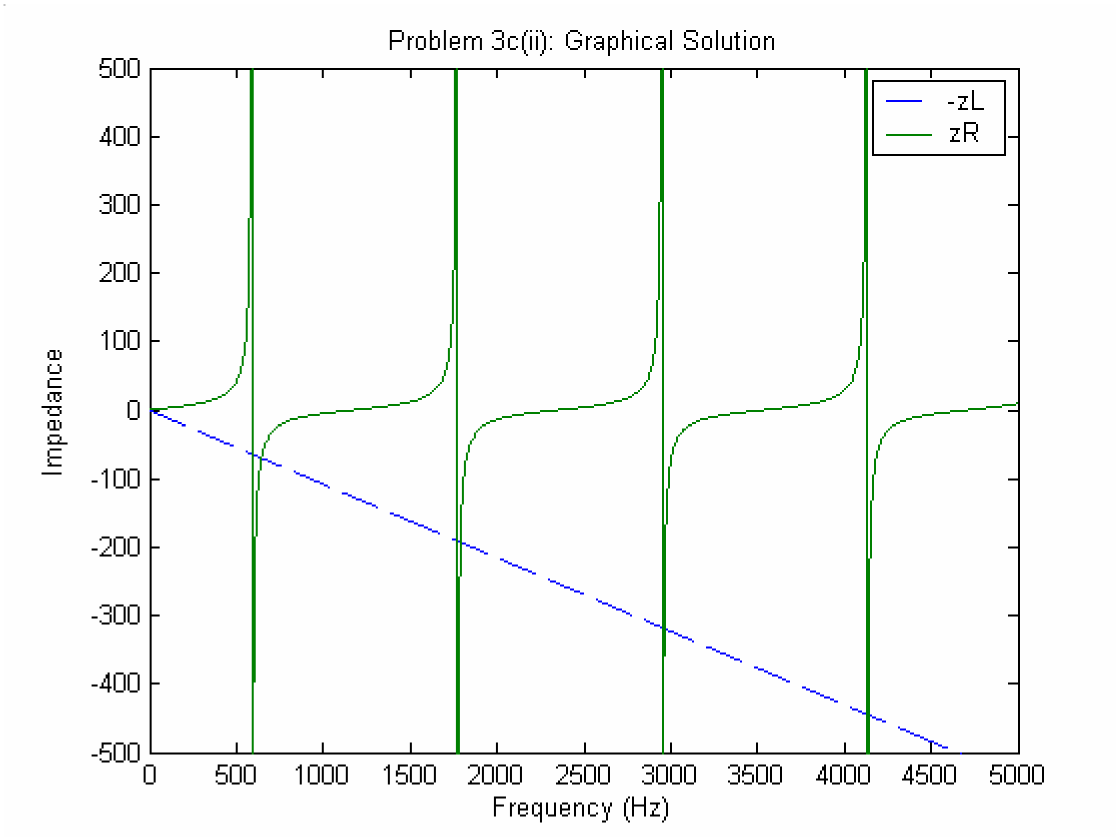
A tube with a new length $l' = l - \Delta l$ = 13.48cm has its lowest natural frequency of $\boxed{F_1' = \frac{c}{4l'} = 656.53 \text{ Hz}}$.

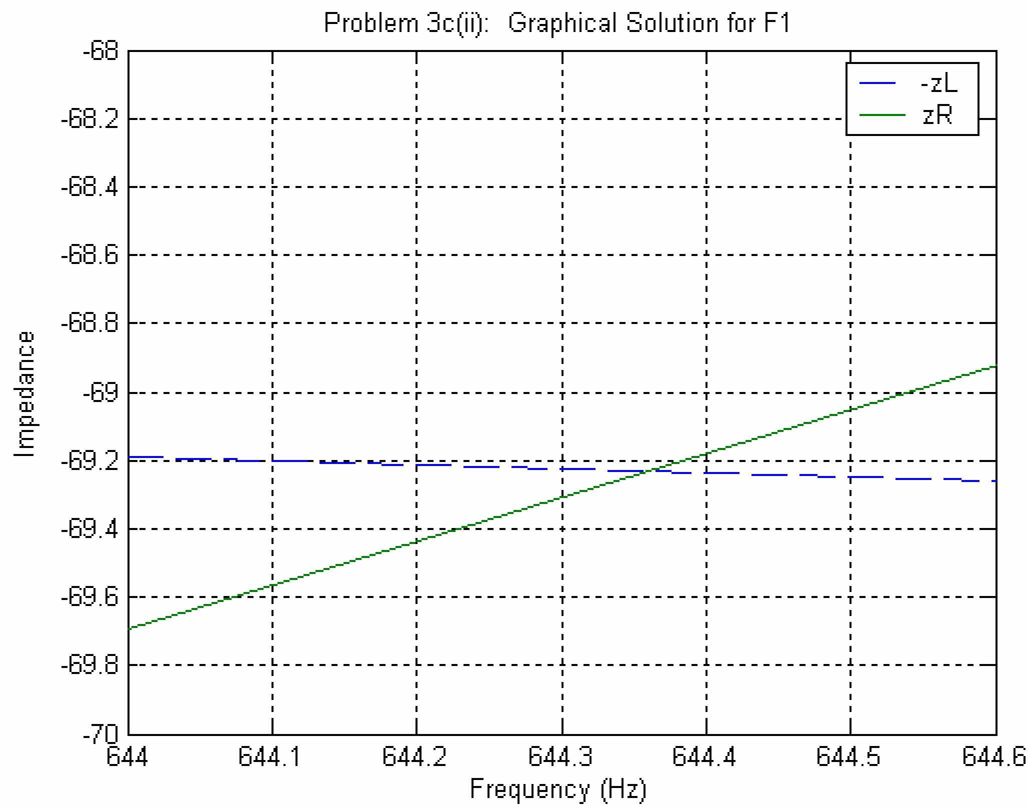
Similarly, setting F = 1770 Hz, the second lowest natural frequency of the original tube , we get Δl = 0.169 cm.

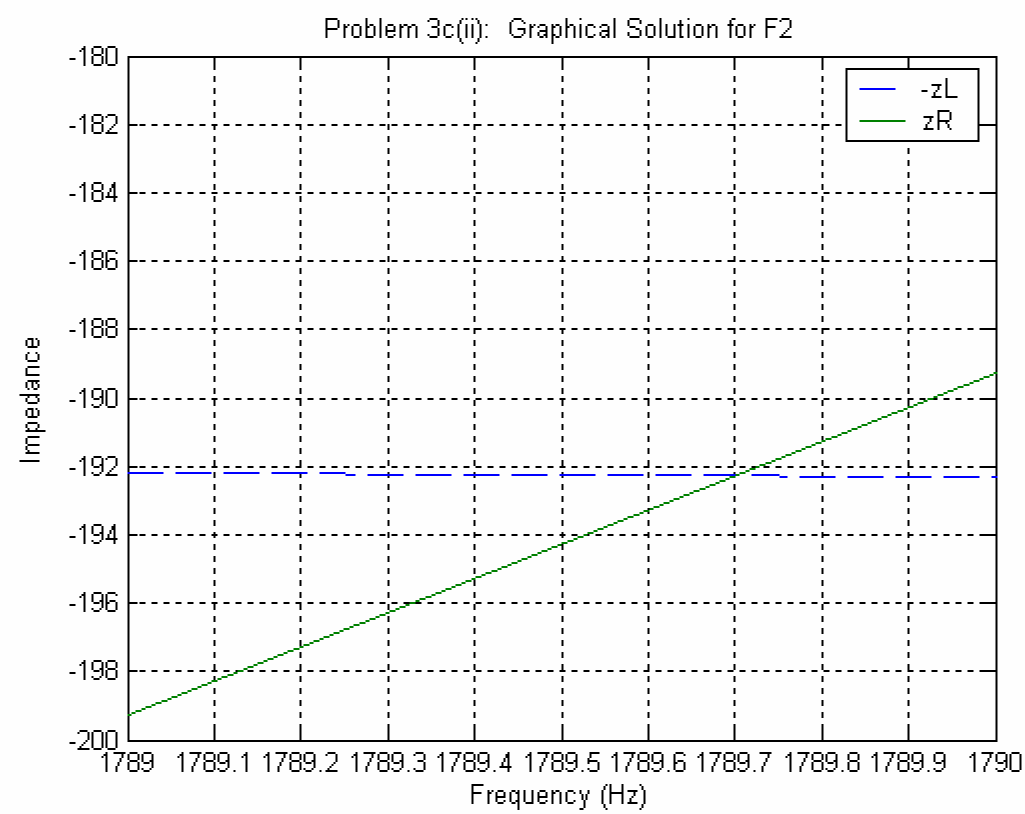
A tube with a new length $l' = l - \Delta l$ = 14.831 cm has its second lowest natural frequency of $\boxed{F_2' = \frac{3c}{4l'} = 1790.17 \text{ Hz}}$.

These are pretty close to the exact solution we get in part (iii).

```
%Problem 3b

%script for calculating the first two resonant frequencies.


eq = inline('(2*pi*x*0.00114*2/2) - (0.00114*35400/4)*cot(2*pi*x*15/35400)');
F1 = fsolve(eq, 591)
F2 = fsolve(eq, 1770)



F1 =

  469.2180

F2 =

   1.4673e+03
```


Problem 3c(ii): Graphical Solution
500
400
300
200
100
0
-100
-200
-300
-400
-500
Impedance
0
500
1000
1500
2000
2500
3000
3500
4000
4500
5000
Frequency (Hz)
-zL
zR


Problem 3c(ii): Graphical Solution for F1
Impedance
-68
-68.2
-68.4
-68.6
-68.8
-69
-69.2
-69.4
-69.6
-69.8
-70
644
644.1
644.2
644.3
644.4
644.5
644.6
Frequency (Hz)
-zL
zR


Problem 3c(ii): Graphical Solution for F2
Impedance
-180
-182
-184
-186
-188
-190
-192
-194
-196
-198
-200
1789
1789.1
1789.2
1789.3
1789.4
1789.5
1789.6
1789.7
1789.8
1789.9
1790
Frequency (Hz)
-zL
zR

```
%Problem 3c(ii)

%This MATLAB script plots the impedances and solve for the natural frequencies
%graphically.

l=15;
A=4;
lc = 0.3;
Ac = 0.02;
rho = 0.00114;
c = 35400;

f = 1:1:5000;

zL = -2*pi*f*rho*lc/Ac;
zR = (rho*c/A)*tan(2*pi*f*l/c);

plot(f,zL, '--', f, zR, '-');


title('Problem 3c(ii): Graphical Solution');
axis([0 5000 -500 500]);
xlabel('Frequency (Hz)');
ylabel('Impedance');
legend('-zL', 'zR');



%zooming in on first resonant frequency

axis([644 644.6 -70 -68])
title('Problem 3c(ii):  Graphical Solution for F1')
grid on;


%zooming in on second resonant frequency
axis([1789 1790 -200 -180])
title('Problem 3c(ii):  Graphical Solution for F2')
```

```
%Problem 3c(iii)
%script for calculating the first two resonant frequencies.


eq = inline('(2*pi*x*0.00114*0.3/0.02) + (0.00114*35400/4)*tan(2*pi*x*15/35400)');
F1 = fsolve(eq, 591)
F2 = fsolve(eq, 1770)



F1 =

  644.3542

F2 =

   1.7897e+03
```