Wednesday, April 20, 2005
Handout #19
Due: Wednesday, April 27, 2005

Problem Set 8

Problem 8.1 (Realizations of repetition and SPC codes)

Show that a reduced Hadamard transform realization of a repetition code RM$(0, m)$ or a single-parity-check code RM$(m-1, m)$ is a cycle-free tree-structured realization with a minimum number of $(3, 1, 3)$ repetition constraints or $(3, 2, 2)$ parity-check constraints, respectively, and furthermore with minimum diameter (distance between any two code symbols in the tree). Show that these two realizations are duals; *i.e.,* one is obtained from the other via interchange of $(3, 2, 2)$ constraints and $(3, 1, 3)$ constraints.

Problem 8.2 (Dual realizations of RM codes)

Show that in general a Hadamard transform (HT) realization of any Reed-Muller code RM(r, m) is the dual of the HT realization of the dual code RM$(m - r - 1, m)$; *i.e.,* one is obtained from the other via interchange of $(3, 2, 2)$ constraints and $(3, 1, 3)$ constraints.

Problem 8.3 (BCJR (sum-product) decoding of SPC codes)

As shown in Problem 6.4, any $(\mu+1, \mu, 2)$ binary linear SPC block code may be represented by a two-state trellis diagram. Let $\mu = 7$, and let the received sequence from a discrete-time AWGN channel be given by $\mathbf{r} = (0.1, -1.0, -0.7, 0.8, 1.1, 0.3, -0.9, 0.5)$. Perform BCJR (sum-product) decoding of this sequence, using the two-state trellis diagram of the $(8, 7, 2)$ SPC code.

Compare the performance and complexity of BCJR decoding to that of the Viterbi algorithm and Wagner decoding (Problem 6.6).