

## CHANNEL MEASUREMENT

Channel measurement doesn't help for single bit transmission in flat Rayleigh fading.

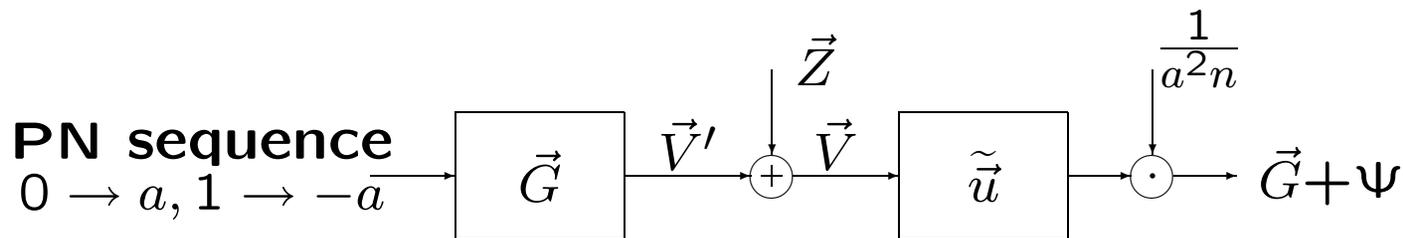
It helps (as we soon see) in detection with multi-tap fading, multiple frequencies, multiple antennas, etc. (i.e., in the presence of diversity).

It helps the transmitter (if known there) to adjust power and rate.

## Pseudonoise (PN) PROBING SIGNALS

A PN sequence  $\vec{u}$  is a binary sequence that appears to have iid components.

A maximal-length binary shift register of  $k$  stages generates all  $2^k - 1$  binary non-zero  $k$ -tuples and is periodic with length  $2^k - 1$ .



$\vec{u}$  is  $\approx$  orthogonal to each shift of  $\vec{u}$  so

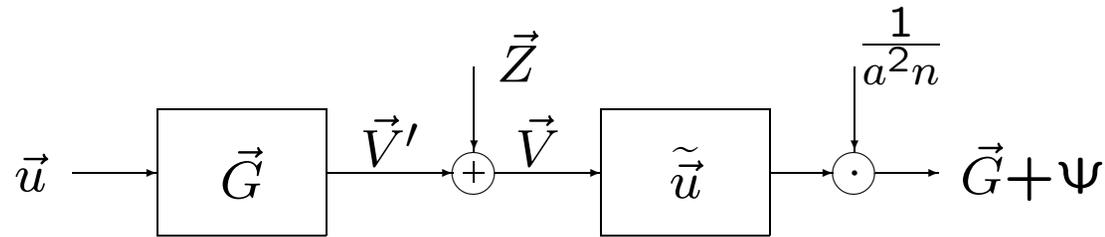
$$\sum_{m=1}^n u_m u_{m+k}^* \approx \begin{cases} a^2n & ; k = 0 \\ 0 & ; k \neq 0 \end{cases} = a^2n \delta_k$$

If  $\vec{u}$  is matched filter to  $\vec{u}$ , then  $\vec{u} * \vec{u} = a^2n \delta_j$ .

If  $\vec{u} * \tilde{\vec{u}} = a^2 n \delta_j$ , then

$$\vec{V}' * \tilde{\vec{u}} = (\vec{u} * \vec{G}) * \tilde{\vec{u}} = (\vec{u} * \tilde{\vec{u}} * \vec{G}) = a^2 n \vec{G}$$

A PN sequence has the same effect as using a single input (of  $n$  times the power) surrounded by zeros.



The response at time  $m$  of  $\tilde{u}$  to  $\vec{Z}$  is the sum of  $n$  iid complex rv's  $\mathcal{N}_c(0, a^2 N_0 W)$ .

The sum has variance  $a^2 n N_0 W$ . After scaling by  $1/(a^2 n)$ ,  $\mathbf{E}[|\Psi_k|^2] = \frac{N_0 W}{a^2 n}$ .

## RAKE RECEIVER

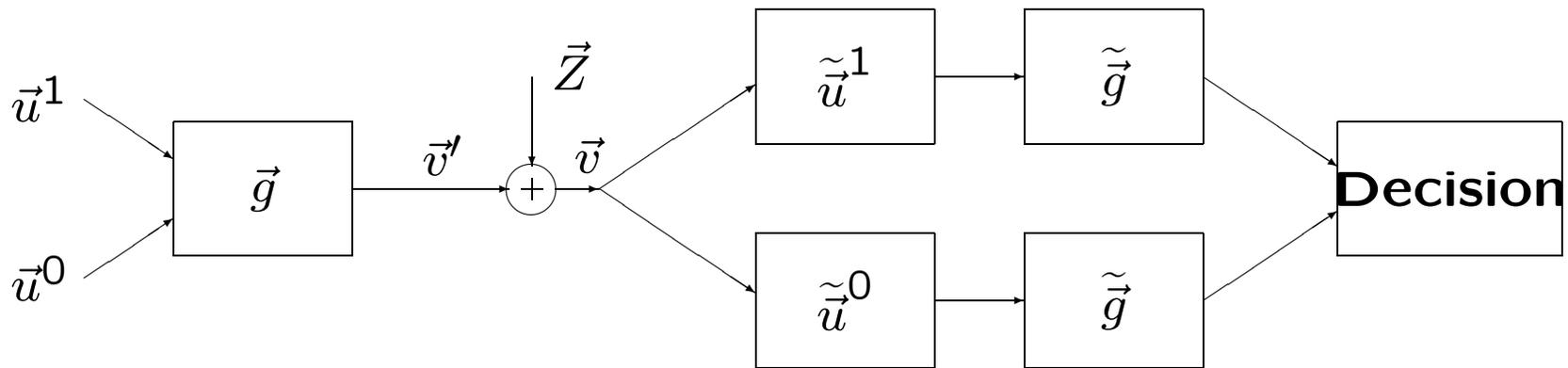
The idea here is to measure the channel and make decisions at the same time.

Assume a binary input,  $H=0 \rightarrow \vec{u}^0$  and  $H=1 \rightarrow \vec{u}^1$

With a known channel  $\vec{g}$ , the ML decision is based on pre-noise inputs  $\vec{u}^0 * \vec{g}$  and  $\vec{u}^1 * \vec{g}$ .

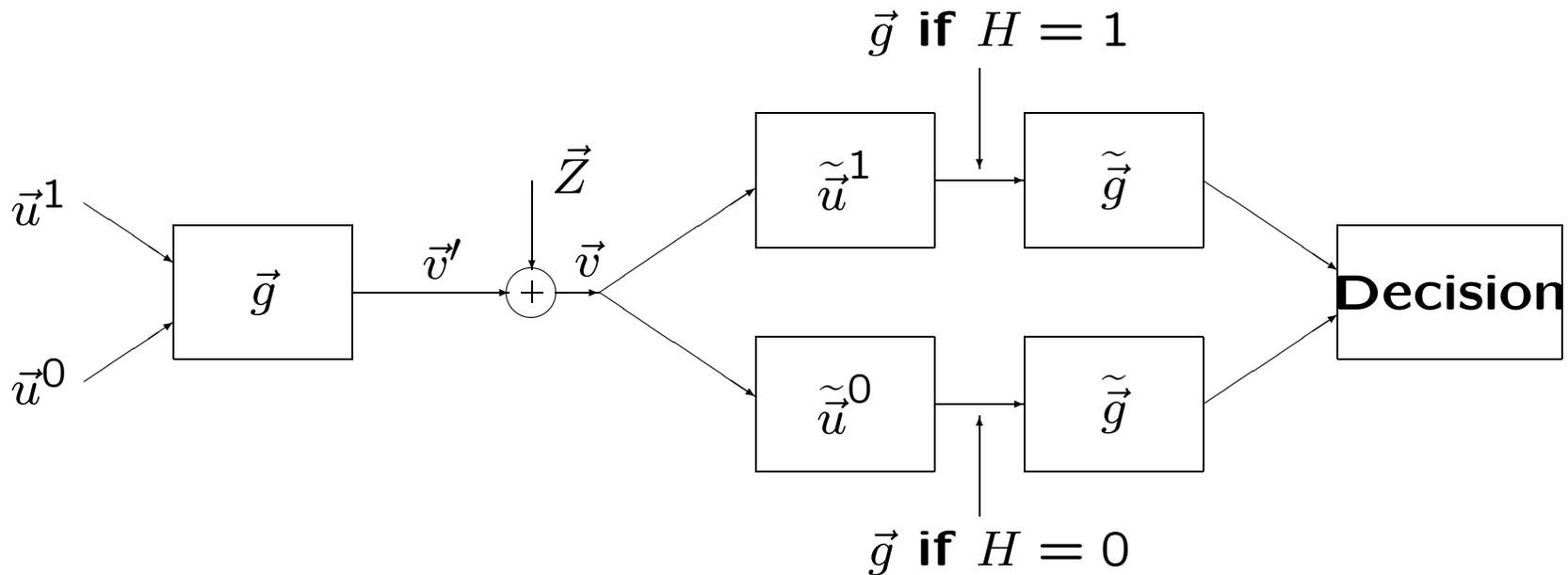
$$\Re(\langle \vec{v}, \vec{u}^0 * \vec{g} \rangle) \underset{\tilde{H}=1}{\overset{\tilde{H}=0}{\gtrless}} \Re(\langle \vec{v}, \vec{u}^1 * \vec{g} \rangle).$$

We can detect using filters matched to  $\vec{u}^0 * \vec{g}$  and  $\vec{u}^1 * \vec{g}$



**Note the similarity of this to the block diagram for measuring the channel.**

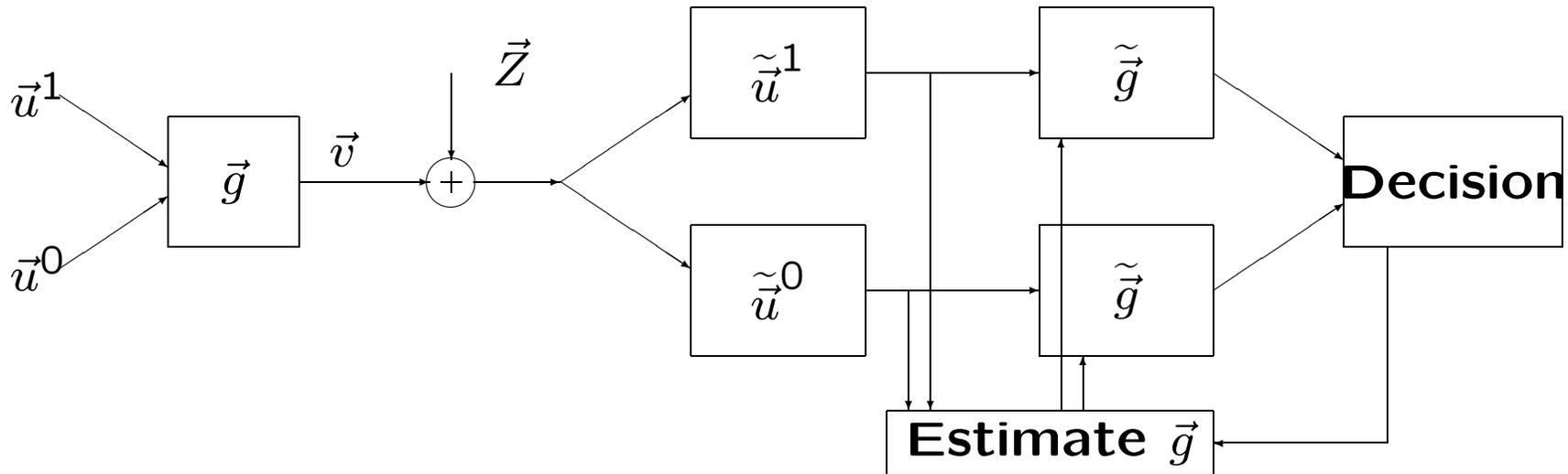
**If the inputs are PN sequences (which are often used for spread spectrum systems), then if the correct decision can be made, the output of the corresponding arm contains a measurement of  $\vec{g}$ .**



$\vec{u}^1$  and  $\vec{u}^0$  are non-zero from time 1 to  $n$ .  $\vec{v}'$  is non-zero from 1 to  $n+k_0-1$ .

$\tilde{u}^1$  and  $\tilde{u}^0$  are non-zero from  $-n$  to  $-1$  (receiver time).

If  $H = 1$  or  $H = 0$ , then  $\vec{g}$  plus noise appears from time 0 to  $k_0 - 1$  where shown. Decision is made at time 0, receiver time.



If  $\hat{H} = 0$ , then a noisy version of  $\vec{g}$  probably exists at the output of the matched filter  $\tilde{\vec{u}}^0$ . That estimate of  $\vec{g}$  is used to update the matched filters  $\tilde{\vec{g}}$ .

If  $\mathcal{T}_c$  is large enough, the decision updates can provide good estimates.

Suppose there is only one Rayleigh fading tap in the discrete-time model.

Suppose the estimation works perfectly and  $\vec{g}$  is always known. Then the probability of error is the coherent error probability  $Q(\sqrt{E_b/N_0})$  for orthogonal signals and  $E_b = a^2 n |g|^2 / W$ .

This is smaller than incoherent  $\Pr(e) = \frac{1}{2} \exp\{-E_b/(2N_0)\}$ .

Averaging over  $G$ , incoherent result is  $\frac{1}{2 + \overline{E_b}/N_0}$  and coherent result is almost the same.

Measurement helps in allowing antipodal transmission.

## Case study: CDMA, IS95

Uses frequency band from 800 to 900 MH.

800-850 used for reverse channel (cell phones → base).

850-900 for forward channel.

Need for considerable separation (45mH) to avoid self noise.

Individual subbands of 1.25 MH.



## Voice compressor

Input voice segmented into 20ms segments.

Each segment compressed independently into 172 bits. This encodes voice into 8.6 kbps.

Old fashioned voice digitization uses 4 kHz. nominal BW (8 ksps), 8 bit quantization, 64 kbps

12 parity checks per segment for error detection.

Another 8 zeros per segment are added to terminate subsequent convolutional code.

Result is 192 bits per 20 ms (9600 bps).

**All timing in encoder and decoder keyed to 9600 bps and 20 ms. intervals.**

**Source/channel separation violated by 20ms.**

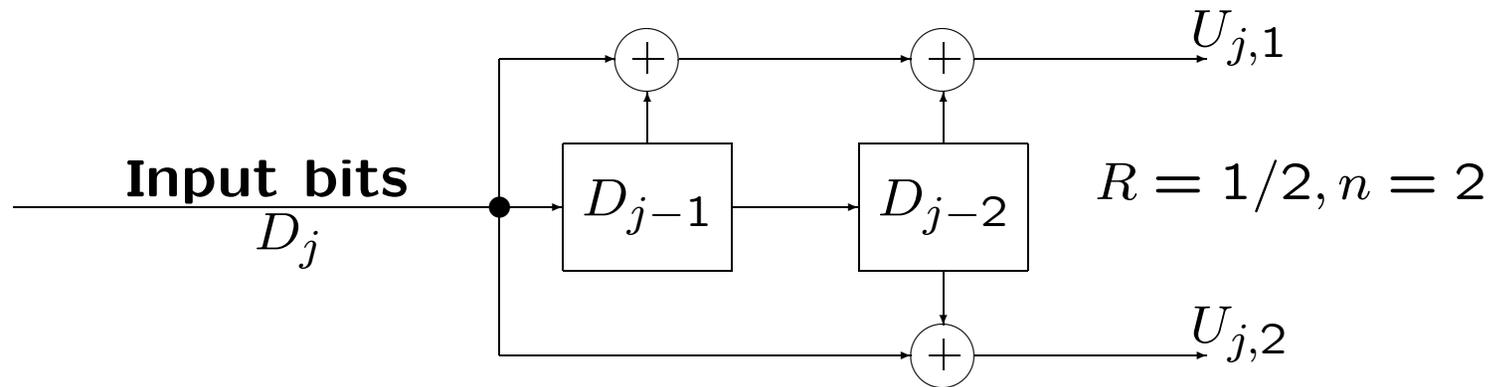
**Caused by the strange nature of voice.**

**Small delay is critical for voice. Larger delay violates conventions for human conversation.**

**Voice has many statistical constraints lasting far longer than 20 ms, but compression can't use them.**

**Channel coding could use longer codewords effectively, but can't in voice systems.**

**A convolutional encoder,  $R = 1/3$ , constraint length  $n = 8$  follows the source compression.**



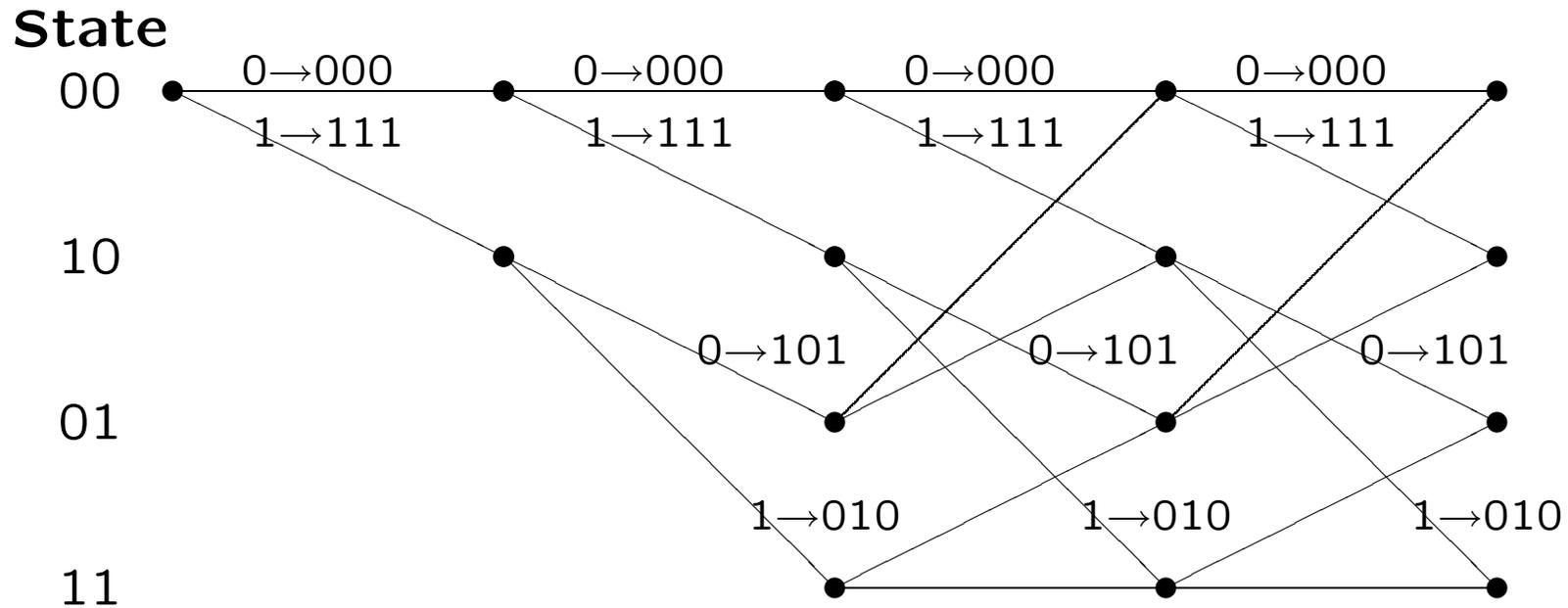
$$U_{j,1} = D_j \oplus D_{j-1} \oplus D_{j-2}$$

$$U_{j,2} = D_j \oplus D_{j-2}$$

**It needs  $n$  bits at end of block to return to state 0.**

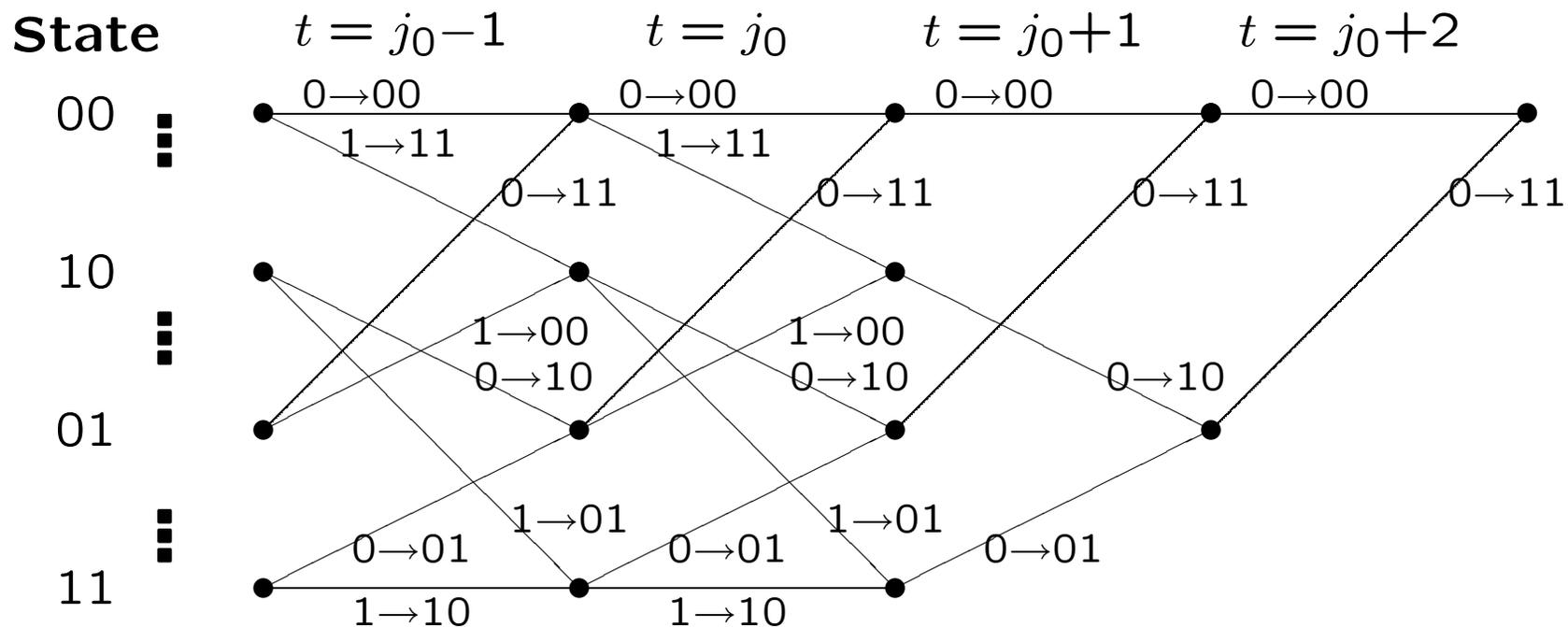
**Viterbi algorithm used for decoding; complexity  $\sim 2^n$ .**

## Trellis for Viterbi decoder



Conventional soft decoder: use log likelihood ratio (against a reference) for each input bit. Sum of LLR's is LLR for a sequence.

The code is terminated with  $n$  bits.



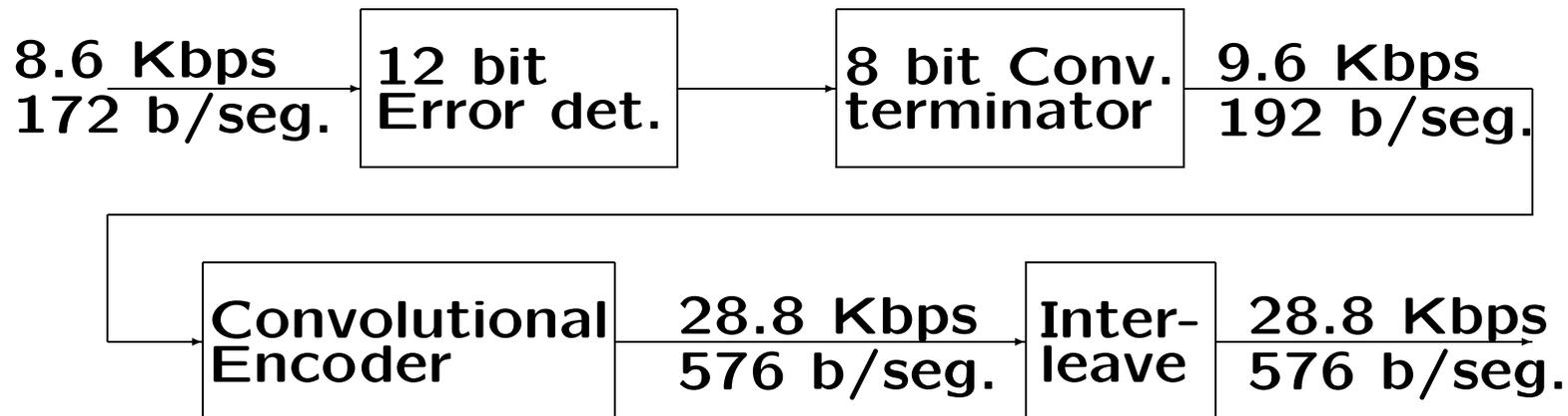
Codeword length is  $3(j_0 + n)$ . The  $n = 8$  termination bits was added to input.

## Voice compression and Channel coding

20 ms voice segments compressed to 172 bits.

20 bit overhead, then rate 1/3 code.

576 bit block output then interleaved



## Modulation by Hadamard matrix

Interleaver output is segmented into 6 bit blocks.

Map 6 bit blocks to 64 bit orthogonal code-words.

0	0
0	1

$b = 1$

0	0	0	0
0	1	0	1
0	0	1	1
0	1	1	0

$b = 2$

0000	0000
0101	0101
0011	0011
0110	0110
0000	1111
0101	1010
0011	1100
0110	1001

$b = 3$

Generate  $H_{b+1}$  from  $H_b$ : put  $H_b$  at top left, top right, lower left, and put complement  $\overline{H_b}$  at lower right.

## Modulation

Segment interleaver output into 6 bit segments.

Map 6 bit  $\rightarrow$  64 bits rows of  $H_6$ .

Map selected row into antipodal signals.

This gives us orthogonal code of length 64.

$28.8 \text{ kbps} \rightarrow (28.8)(64)/6 = 307.2 \text{ kbps}$

Receiver uses these codewords in a rake receiver.

Rake has 64 branches rather than 2.

Incoherent decisions are used.

**For orthogonal code, incoherent reception in WGN**

$$\Pr(e) \leq \frac{63}{2} \exp \left[ \frac{-E_s}{2N_0} \right]$$

$E_s = 6E_b$ , so  $\Pr(e) \leq \frac{63}{2} e^{-3E_b/N_0}$ .

**Effect of fading is to change  $E_b$  over time.**

**Because of multitap diversity,  $E_b$  changes less than for Rayleigh distribution.**

**Typical case:  $E_b$  large, little penalty for incoherent.**

Output of orthogonal decoder could in principle produce 63 LLR's. Want only one LLR for each of 6 bits.

Want  $\log \left( \frac{\text{Pr}(\text{correct})}{\text{Pr}(\text{error})} \right)$

Visualize comparing two codewords 0000000 and 1111111 for hard decisions with error probability  $p$ .

If errors are independent,  $P_e \approx 35p^4$ . If perfectly correlated,  $P_e = p \gg 35p^4$  for  $p$  small.

Lesson: Highly dependent errors screw up decoding.

Solution: Interleaving.

## Rest of modulator

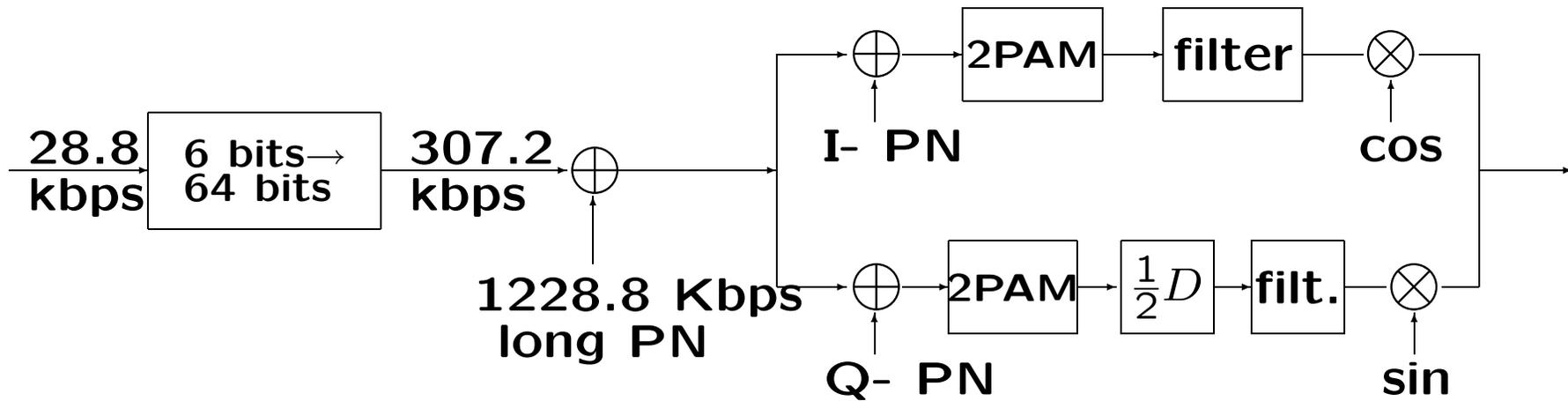
The sequence of Walsh functions from encoder are multiplied by PN sequence at 4 times 307.2 kbps (1.2288mbps)

This is the output of a 42 bit linear shift register.

The period is  $2^{42} - 1$ , with this many possible starting states.

Each cell phone starts with a different state in this PN sequence. Only difference between different cell phones.

This Mod-2 addition on Hadamard output doesn't effect orthogonality.



The I-PN and Q-PN are used to make the inphase and quadrature components look different.

How can this be received without a phase lock loop?

Why is incoherent reception necessary then?

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