Review of multipath wireless model

The response to $\exp[2\pi i f t]$ **over** J **propagation paths with attenuation** β_j **and delay** $\tau_j(t)$ **is**

$$\begin{aligned} y_f(t) &= \sum_{j=1}^{J} \beta_j \exp[2\pi i f t - \tau_j(t)] \\ &= \hat{h}(f,t) \exp[2\pi i f t] \end{aligned}$$

The response to $x(t) = \int_{-\infty}^{\infty} \hat{x}(f) \exp[2\pi i f t]$ **is then**

$$\begin{aligned} y(t) &= \int_{-\infty}^{\infty} \hat{x}(f) \hat{h}(f,t) \exp(2\pi i f t)\, df \\ &= \int x(t-\tau)\, h(\tau,t)\, d\tau \quad \textbf{where} \end{aligned}$$

$$h(\tau,t) \longleftrightarrow \hat{h}(f,t); \qquad h(\tau,t) = \sum_j \beta_j \delta\{\tau - \tau_j(t)\}$$

How do we define fading for a single frequency input?

$$
\begin{aligned}
y_f(t) &= \hat{h}(f,t)\,\mathsf{exp}[2\pi i f t] \\
&= |\hat{h}(f,t)|\,\mathsf{exp}[2\pi i f t + i\angle \hat{h}(f,t)] \\
\Re[y_f(t)] &= |\hat{h}(f,t)|\cos[2\pi f t + \angle \hat{h}(f,t)]
\end{aligned}
$$

The envelope of this is $|\hat{h}(f,t)|$, and this is defined as the fading.

$$
\hat{h}(f,t) = \sum_j \beta_j \,\mathsf{exp}[-2\pi i f \tau_j(t)] = \sum_j \mathsf{exp}[2\pi i \mathcal{D}_j t - 2\pi i f \tau_j^o]
$$

This contains frequencies ranging from $\min \mathcal{D}_j$ to $\max \mathcal{D}_j$. Define the Doppler spread of the channel as

$$
\mathcal{D} = \max \mathcal{D}_j - \min \mathcal{D}_j
$$

For any frequency Δ, $|\hat{h}(f,t)| = |e^{-2\pi i \Delta t}\hat{h}(f,t)|$

$$\hat{h}(f,t) = \sum_j \exp\{2\pi i \mathcal{D}_j t - 2\pi i f \tau_j^o\}$$

Choose $\Delta = [\max \mathcal{D}_j + \min \mathcal{D}]/2$**. Then**

$$\exp(-2\pi i t\Delta)\,\hat{h}(f,t) = \sum_{j=1}^{J} \beta_j \exp\{2\pi i t(\mathcal{D}_j - \Delta) - 2\pi i f \tau_j^o\}$$

This waveform is baseband limited to $\mathcal{D}/2$**. Its magnitude is the fading. The fading process is the magnitude of a waveform baseband limited to** $\mathcal{D}/2$**. The coherence time of the channel is defined as**

$$\mathcal{T}_{\text{coh}} = \frac{1}{2\mathcal{D}}$$

$\mathcal{D}$ **is linear in** f**;** $\mathcal{T}_{\text{coh}}$ **goes as** $1/f$**.**

$\mathcal{D}$ and $\mathcal{T}_{\text{coh}}$ are two of the primary characteristics of a multipath channel. The other two are

$$\mathcal{L} \;=\; \max_j \tau_j(t) - \min \tau_j(t) \quad \textbf{and } \mathcal{F}_{\text{coh}} = \frac{1}{2\mathcal{L}}$$

$\mathcal{F}_{\text{coh}}$ is the change in carrier frequency required for the fading to change substantially. This is essentially the T/F dual of the relationship between $\mathcal{T}_{\text{coh}}$ and $\mathcal{D}$.

$$\exp[2\pi i f \tau_{\text{mid}}]\, \hat{h}(f,t) = \sum_j \beta_j \exp\{-2\pi i f[\tau_j(t) - \tau_{\text{mid}}]\}$$

The quantity on the right, as a function of f, is "baseband limited" to $\mathcal{L}/2$.

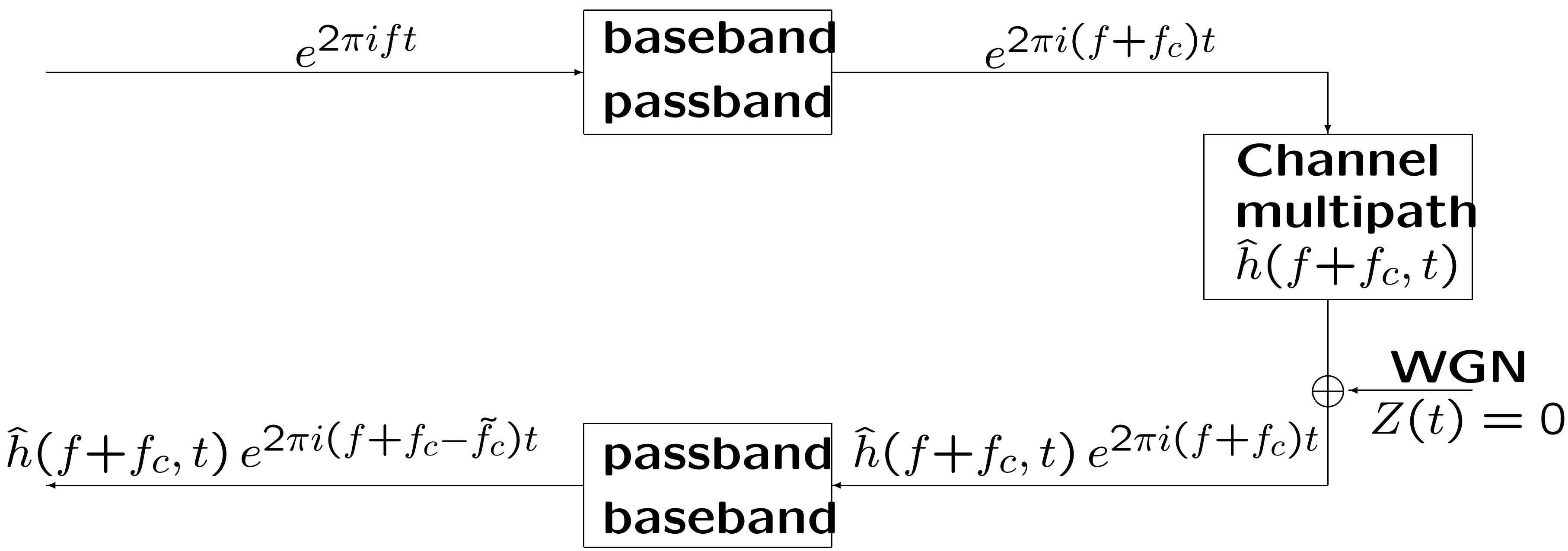
$$\exp[2\pi i f \tau_{\text{mid}}]\, \hat{h}(f,t) = \sum_j \beta_j \exp\{-2\pi i f [\tau_j(t) - \tau_{\text{mid}}]\}$$

The magnitude of the quantity on the left is the fading at f and t.

The fading in frequency changes significantly over 1/4 the bandwidth on the right, ($\mathcal{L}/2$); $\mathcal{F}_{\text{coh}}$ is the order of magnitude change in f over which the fading changes.

The timing recovery at the receiver tends to keep τ_{mid} close to 0.

Baseband system functions



Let $\Delta = \tilde{f}_c - f_c$ be the frequency offset in demodulation. Let $\hat{g}(f,t) = \hat{h}(f+f_c,t)e^{-2\pi i\Delta t}$.

Then $\hat{g}(f,t)$ is the baseband system function.

Ignoring the noise for now, the response to a complex baseband input $u(t)$ is

$$\begin{aligned} v(t) &= \int_{-W/2}^{W/2} \hat{u}(f)\hat{h}(f+f_c,t)\, e^{2\pi i(f-\Delta)t}\, df \\ &= \int_{-W/2}^{W/2} \hat{u}(f)\, \hat{g}(f,t)\, e^{2\pi i f t}\, df \end{aligned}$$

By the same relationship between frequency and time we used for bandpass,

$$v(t) = \int_{-\infty}^{\infty} u(t-\tau)g(\tau,t)\, d\tau$$

where $g(\tau,t)$ is the inverse Fourier transform (for fixed t) of $\hat{g}(f,t)$.

For the simplified multipath multipath model, $\hat{h}(f,t) = \sum_{j=1}^{J} \beta_j \exp\{-2\pi i f \tau_j(t)\}$ **and thus the baseband system function is**

$$\hat{g}(f,t) = \sum_{j=1}^{J} \beta_j \exp\{-2\pi i (f + f_c)\tau_j(t) - 2\pi i \Delta t\}$$

We can separate the dependence on t **from that on** f **by rewriting this as**

$$\hat{g}(f,t) = \sum_{j=1}^{J} \gamma_j(t) \exp\{-2\pi i f \tau_j(t)\} \qquad \textbf{where}$$

$$\gamma_j(t) = \beta_j \exp\{-2\pi i f_c \tau_j(t) - 2\pi i \Delta t\}$$

For the ray tracing model, $\widehat{h}(f,t) = \sum_j \beta_j(t)\,\mathsf{exp}\{-2\pi i f \tau_j(t)\}$**.**

$$\widehat{g}(f,t) = \sum_j \beta_j(t)\,\mathsf{exp}\{-2\pi i (f + f_c)\tau_j(t)\}$$

$$g(\tau,t) = \sum_j \beta_j(t)\,\mathsf{exp}\{-2\pi i f_c \tau_j(t)\}\,\delta[\tau - \tau_j(t)]$$

$$v(t) = \sum_j \beta_j(t)\,\mathsf{exp}\{-2\pi i f_c \tau_j(t)\}\,u[t - \tau_j(t)]$$

In terms of Doppler shifts,

$$v(t) = \sum_j \beta_j(t)\,\mathsf{exp}\{-2\pi i (f_c \tau_j^o - \mathcal{D}_j t)\}\,u[t - \tau_j(t)]$$

The recovered carrier f_c' **will be shifted to compensate for systematic Doppler shifts. Thus the shifts relative to the recovered carrier will**

lie roughly in the range $\pm\mathcal{D}/2$**. Thus** $\mathcal{T}_c = 1/(2\mathcal{D})$**.**

Discrete-time baseband model

$$u(t) = \sum_n u_n \mathrm{sinc}(Wt - n)$$

$$\begin{aligned} v(t) &= \sum_j \beta_j(t) \exp\{2\pi i f_c \tau_j(t)\} u(t - \tau_j(t)) \\ &= \sum_n u_n \sum_j \beta_j(t) \exp\{-2\pi i f_c \tau_j(t)\} \mathrm{sinc}[W(t - \tau_j(t)) - n] \end{aligned}$$

The sampled outputs at multiples of $T = 1/W$, i.e. $v_m = v(mT)$ are then given by

$$v_m = \sum_n u_n \sum_j \beta_j(mT) \exp\{-2\pi i f_c \tau_j(mT)\} \mathrm{sinc}[m - n - \tau_j(mT)/T]$$

$$\begin{aligned} v_m &= \sum_k u_{m-k} \sum_j \beta_j(mT) \exp\{-2\pi i f_c \tau_j(mT)\} \mathrm{sinc}[k - \tau_j(mT)/T] \\ &= \sum_k u_{m-k}\, g_{k,m} \qquad \textbf{where } k = m - n. \end{aligned}$$

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