

Wireless Systems are now digital (Binary Interface)

Source either analog or digital.

Cellular systems developed for voice: need small fixed delay; fixed data rate; not high reliability.

Data: delay less important, reliability more important; variable but high data rate.

OTHER WIRELESS SYSTEMS:

Broadcast Systems;

Wireless LANs (often in home or office);

Adhoc Networks.

Standardization is a major problem for all wireless systems, particularly with roaming.

Will cellular and LANS merge into one network?

What is the market for high speed mobile data?

We study more technical issues here.

PHYSICAL MODELING

Cellular systems use bandwidths from several KH to MH around GH carriers.

Cellular ranges are small, a few KM or less.

Narrow band; WGN assumption good, but new problems are fading and interference.

EM equations are too difficult to solve and constantly changing.

Very different modeling questions arise in the placement of base stations from those in the design of mobiles and base stations.

Look at idealized models for clues.

Consider fixed antenna in free space:

Response at $\mathbf{d} = (r, \theta, \psi)$ to $\cos(2\pi ft)$ is:

$$E(f, t, \mathbf{d}) = \frac{1}{r} \Re \left[\alpha_s(\theta, \psi, f) \exp\left\{2\pi i f \left(t - \frac{r}{c}\right)\right\} \right]$$

Note $1/r$ attenuation; think spheres.

Receiving antenna alters field; doesn't depend on $\mathbf{d} = (r, \theta, \psi)$. The received field at \mathbf{d} is then

$$E_r(f, t, \mathbf{d}) = \frac{\Re [\alpha(\theta, \psi, f) \exp\{2\pi i f(t - r/c)\}]}{r}$$

where α takes into effect both the transmit antenna and the effect of the receive antenna.

Define the system function for the receiver at \mathbf{d} :

$$\hat{h}(f) = \frac{\alpha(\theta, \psi, f) \exp\{-2\pi i f r / c\}}{r}.$$

Then the response to $\exp(2\pi i f t)$ is

$$E_r(f, t, \mathbf{d}) = \hat{h}(f) \exp\{2\pi i f t\}$$

The system is LTI because Maxwell's equations are linear and nothing is moving.

The response to an arbitrary input $x(t) = \int \hat{x}(f) \exp\{2\pi i f t\} df$ is then

$$y(t) = \int_{-\infty}^{\infty} \hat{x}(f) \hat{h}(f) \exp\{2\pi i f t\} df.$$

$$y(t) = \int_{-\infty}^{\infty} \hat{x}(f)\hat{h}(f) \exp\{2\pi i f t\} df.$$

Define $h(t) = \int \hat{h}(f) \exp(2\pi i f t) df$. **Then**

$$y(t) = \int x(\tau)h(t - \tau) d\tau$$

If α is real and independent of f , then

$$h(t) = \frac{\alpha}{r} \delta\left(t - \frac{r}{c}\right)$$

$$y(t) = \frac{\alpha}{r} x\left(t - \frac{r}{c}\right)$$

Since $\hat{y}(f) = \hat{x}(f)\hat{h}(f)$, this only requires α to be independent of f over the bandwidth of x .

Summary: For two fixed antennas in free space, the output is the same as input, attenuated by α/r and delayed by r/c .

Next assume the receiving antenna is moving away from transmitter with velocity v . Thus $r(t) = r_0 + vt$. The field at $r(t)$ in the absence of a receiving antenna is

$$E(f, t, \mathbf{d}(t)) = \frac{\alpha_s(\theta, \psi, f) \exp\{2\pi i f(t - r_0/c - vt/c)\}}{r_0 + vt}.$$

Putting the receiving antenna in, the response at the antenna terminals is

$$\frac{\alpha(\theta, \psi, f) \exp\{2\pi i [f(1 - \frac{v}{c})t - \frac{fr_0}{c}]\}}{r_0 + vt},$$

$$\frac{\alpha(\theta, \psi, f) \exp\{2\pi i[f(1-\frac{v}{c})t - \frac{fr_0}{c}]\}}{r_0 + vt},$$

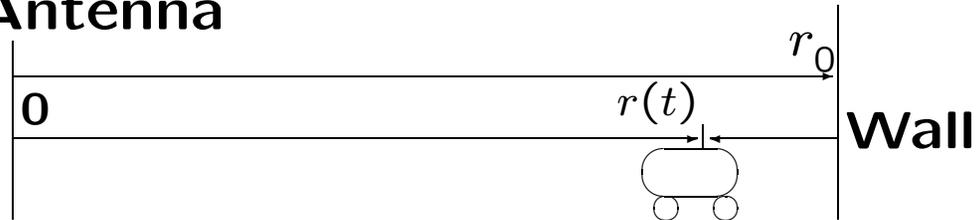
This is a sinusoid at frequency $f(1-v/c)$. There has been a Doppler shift of fv/c .

The system is still linear but not time-invariant. It is linear-time-varying (LTV).

This does not hamper communication; the carrier recovery operates at the shifted carrier.

Moving antenna, reflecting wall

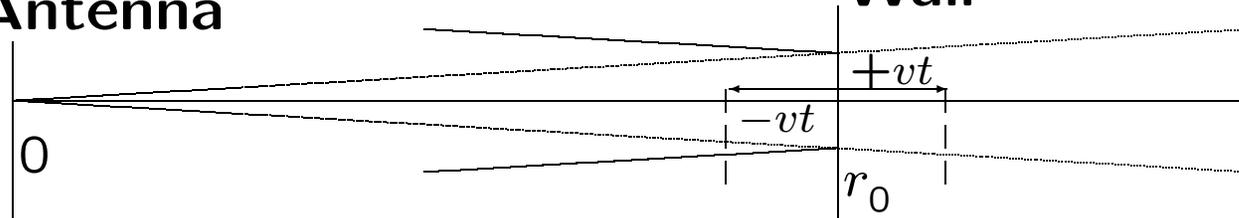
Sending Antenna



60 km/hr

$$r(t) = r_0 - vt$$

Sending Antenna



Reflected wave from right with path length $r_0 + vt$

Direct wave from left with path length $r_0 - vt$

Assume the receiving antenna has a uniform response. The received waveform in response to $\exp(2\pi if t)$ is then

$$y_f(t) = \frac{\alpha \exp\{2\pi if [t - \frac{r_0 - vt}{c}]\}}{r_0 - vt} - \frac{\alpha \exp\{2\pi if [t - \frac{r_0 + vt}{c}]\}}{r_0 + vt}.$$

The direct wave has a positive Doppler shift; the reflected wave has a negative Doppler shift.

When t is small, both sinusoids have about the same amplitude, but one is at a slightly higher frequency.

They alternate between reinforcing and cancelling each other.

$$y_f(t) = \frac{\alpha \exp\{2\pi i f [t - \frac{r_0 - vt}{c}]\}}{r_0 - vt} - \frac{\alpha \exp\{2\pi i f [t - \frac{r_0 + vt}{c}]\}}{r_0 + vt}.$$

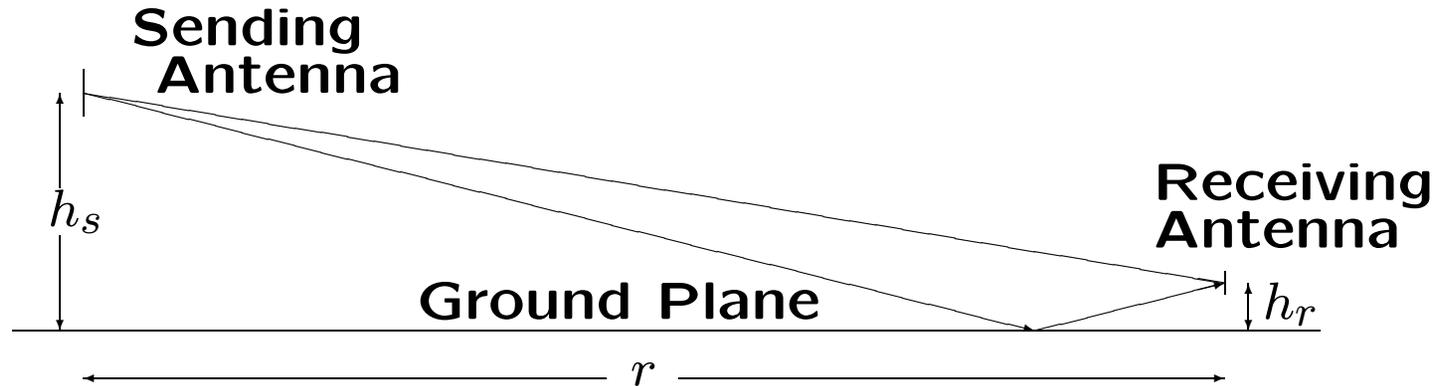
Assuming t small and $r_0 \approx r_0 + vt \approx r_0 - vt$,

$$\begin{aligned} y_f(t) &\approx \frac{\alpha \exp\{2\pi i f [t - \frac{r_0}{c}]\} (e^{2\pi i f vt/c} - e^{-2\pi i f vt/c})}{r_0} \\ &= \frac{2i \alpha \exp\{2\pi i f [t - \frac{r_0}{c}]\} \sin\{2\pi f vt/c\}}{r_0}. \end{aligned}$$

The Doppler shift frequency is modulating the transmitting sinusoid. At $v = 60$ kmH and $f = 1$ gH, $fv/c = 50$ H.

This is multipath fading; 1 fade each 10 ms.

Sometimes multiple reflections don't create multipath; consider reflection from a ground plane:



The direct path and reflecting path differ in length by an amount proportional to $1/r$.

The two received electromagnetic waves are opposite in sign with a phase difference $\sim 1/r$.

Thus the power attenuation over a ground plane is $\sim 1/r^4$.

Input /Output models for wireless

Suppose we have J significant propagation paths.
The response to $e^{2\pi if t}$ on the j th path is

$$\frac{\alpha_j \exp\{2\pi if[t - \frac{r_j(t)}{c}]\}}{r_j(t)}.$$

$$\begin{aligned} y_f(t) &= \sum_{j=1}^J \frac{\alpha_j \exp\{2\pi if[t - \frac{r_j(t)}{c}]\}}{r_j(t)} \\ &= \sum_{j=1}^J \beta_j(t) \exp\{2\pi if[t - \tau_j(t)], \end{aligned}$$

where

$$\beta_j(t) = \frac{\alpha_j(t)}{r_j(t)} \qquad \tau_j(t) = \frac{r_j(t)}{c}$$

Define a linear-time-varying system function $\hat{h}(f, t)$ as the response to $\exp(2\pi i f t)$. Thus

$$\begin{aligned}y_f(t) &= \sum_{j=1}^J \beta_j(t) \exp\{2\pi i f [t - \tau_j(t)]\} \\ &= \exp(2\pi i f t) \hat{h}(f, t) \\ \hat{h}(f, t) &= \sum_{j=1}^J \beta_j(t) \exp\{-2\pi i f \tau_j(t)\}\end{aligned}$$

This gives the response to a sinusoidal input, but, as opposed to LTI system functions, it is a function of t .

Most of the time, we use an even simpler model where β_j does not vary in time and where $\tau_j(t)$ changes at a constant rate (i.e., motion at a constant velocity). Then,

$$\hat{h}(f, t) = \sum_{j=1}^J \beta_j \exp\{-2\pi i f \tau_j(t)\}; \quad \tau_j(t) = \tau_j^0 + \tau_j' t.$$

For reflecting wall,

$$\beta_1 = \frac{\alpha_1}{r_0} \quad \beta_2(t) = \frac{\alpha_2}{r_0}$$

$$\tau_1(t) = \frac{r_0 - vt}{c} \quad \tau_2(t) = \frac{r_0 + vt}{c}$$

In general, for an LTV system, the response to $\exp(2\pi i f t)$ is $\hat{h}(f, t) \exp\{2\pi i f t\}$. By linearity, the response to $x(t)$ is

$$y(t) = \int_{-\infty}^{\infty} \hat{x}(f) \hat{h}(f, t) \exp\{2\pi i f t\} df$$

But we can not argue that $\hat{y}(f) = \hat{x}(f) \hat{h}(f, t)$.

This would make $\hat{y}(f)$ a function of t .

Also the response to a sinusoid is spread in frequency.

However, there is a nice analog to LTI convolution.

$$y(t) = \int_{-\infty}^{\infty} \hat{x}(f) \hat{h}(f, t) \exp\{2\pi i f t\} df$$

$$h(\tau, t) = \int_{-\infty}^{\infty} \hat{h}(f, t) \exp(2\pi i f \tau) df$$

$$\hat{h}(f, t) = \int_{-\infty}^{\infty} h(\tau, t) \exp(-2\pi i f \tau) d\tau.$$

$$\begin{aligned} y(t) &= \int_{-\infty}^{\infty} \hat{x}(f) \left[\int_{-\infty}^{\infty} h(\tau, t) \exp[2\pi i f(t - \tau)] d\tau \right] df \\ &= \int_{-\infty}^{\infty} h(\tau, t) \left[\int_{-\infty}^{\infty} \hat{x}(f) \exp[2\pi i f(t - \tau)] df \right] d\tau \end{aligned}$$

$$y(t) = \int_{-\infty}^{\infty} \hat{x}(f) \left[\int_{-\infty}^{\infty} h(\tau, t) \exp[2\pi i f(t - \tau)] d\tau \right] df$$

$$h(\tau, t) = \int_{-\infty}^{\infty} \hat{h}(f, t) \exp(2\pi i f\tau) df$$

$$\hat{h}(f, t) = \int_{-\infty}^{\infty} h(\tau, t) \exp(-2\pi i f\tau) d\tau.$$

Interchanging the order of integration,

$$y(t) = \int_{-\infty}^{\infty} h(\tau, t) \left[\int_{-\infty}^{\infty} \hat{x}(f) \exp[2\pi i f(t - \tau)] df \right] d\tau$$

$$y(t) = \int_{-\infty}^{\infty} h(\tau, t) x(t - \tau) d\tau$$

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