6.450 Principles of Digital Communication — Due Mon, Sep 21, at beginning of class
MIT, Fall 2009

Problem Set 1

Problem 1- Problem 2.14 from Gallager's book.

Problem 2- In class, we proved Kraft inequality by mapping each codeword to a rational number in the interval $[0, 1)$. In this problem, we want to show it by using its corresponding binary tree. Suppose $\mathcal{C}$ is our codebook with j codewords, each with length l_j, respectively.

a) When is $\mathcal{C}$ prefix free? (express your answer in terms of properties of a corresponding binary tree)

b) Suppose G is a corresponding binary tree of this codebook. Let $M = \max_j(l_j)$. If G was a complete binary tree with depth M, how many leaves would it have? How many children does a node in depth l_j have in the M^{th} stage of this tree?

c) By using (a) and (b), try to prove Kraft inequality.

Problem 3- Suppose $\mathbf{X}^n$ is a string of n iid binary discrete random symbols $\{X_k : 1 \leq k \leq n\}$, and n is large enough,

a) If $Pr(X_k = 0) = 1/3$ and $Pr(X_k = 1) = 2/3$, what is the entropy of the random variable X_k? What fraction of whole sequences with length n are typical? Determine these sequences.

b) If $Pr(X_k = 0) = Pr(X_k = 1) = 1/2$, how many sequences are typical? Find these typical sequences. Intuitively, in each sequence with length n, how many ones and zeros do you expect? Do all typical sequences have this property? Explain.

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