Massachusetts Institute of Technology

6.435 System Identification

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Problem Set No. 3 — Due 3/16/1994

Reading: Chapter 4, Chapter 7. The homework does not require ch 7.

Problem 1:

Given the ARX model

$$A(q)y = B(q)u + e$$

where A is a stable polynomial (in q^{-1}) of degree na, $A(0) = 1$, and B is a polynomial of degree nb. We would like to study the relations of persistence of excitation as we studied them in class, with the more useful definitions in terms of the regressor vector. Express the output as

$$y = \phi^T(t)\theta + e$$

Let $R = E\phi(t)\phi^T(t)$. For the noise free case, prove that the matrix R is positive definite if and only if u is persistently exciting with order $na + nb$ and $A(z), B(z), z = q^{-1}$ are coprime.

Problem 2

If e in Problem 1 is white noise, uncorrelated with u. Show that $R > 0$ if and only if u is persistently exciting with order nb.

Problem 3

Consider the linear regression model

$$y(t) = a + bt + e(t)$$

Find the least squares estimates of a, b. Treat the following cases:

1. The data are $y(1), y(2), \dots y(N)$.
2. The data are $y(-N), y(-N+1), \dots y(N)$.
3. Let $e(t)$ be WN, with variance λ^2. Find the variance of the quantity $s(t) = \hat{a} + \hat{b}t$. Do this for part 1 only.

4. Write the covariance matrix of $\hat{\theta} = (\hat{a} \quad \hat{b})^T$ in the form

$$P = cov(\hat{\theta}) = \begin{pmatrix} \sigma_1^2 & \rho\sigma_1\sigma_2 \\ \rho\sigma_1\sigma_2 & \sigma_2^2 \end{pmatrix}$$

Find the asymptotic value of the correlation coefficient ρ as N tends to infinity.

Problem 4

Do the following problems from Ljung's book:

a. 4G.2

b. 4G.3

c. 4G.4

d. 4G.8

e. 4E.6