

Recitation 4 Outline

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Binary Hypothesis Tests: Receiver Operating Characteristic (ROC)

1. Partially Singular Hypothesis Test

$H_0 : \mathbf{y}$ uniformly distributed on $[-1, 1]$

$H_1 : \mathbf{y}$ uniformly distributed on $[0, 2]$

- Derivation of ROC: discrete points achievable using likelihood ratio tests
 - Achieve other points on ROC using randomized tests
 - Randomization provides the optimal decision rule under the Neyman-Pearson criterion
2. Performance analysis: given ROC & threshold minimizing probability of error, find
- Prior probabilities
 - Minimum achievable probability of error
 - Achievable tests given MPE rule and a fair coin
 - Given additional assumptions, the likelihood function for each hypothesis

Geometry of M-ary Hypothesis Tests

1. Decomposition to sets of pairwise comparisons
2. Gaussian distributions with different means, shared covariance
 - Minimum distance rule
 - Voronoi diagram construction by intersection of halfspaces
3. Geometry on the conditional probability simplex (§2.8.4)
 - Decision boundaries are hyperplanes defined by costs
 - Visualization of these boundaries when $M=2,3$