Massachusetts Institute of Technology
Department of Electrical Engineering and Computer Science

6.432 Stochastic Processes, Detection and Estimation

Recitation 2 Outline

February 11, 2004

Diagonalization of Symmetric Matrices

1. Review: eigenvalues, eigenvectors, and diagonalization
2. Proof: symmetric matrices have real eigenvalues
3. Proof: symmetric matrices have a complete orthogonal set of eigenvectors
   - Simple proof for distinct eigenvalues
   - Brief introduction to Householder reflections
   - Induction proof valid for repeated eigenvalues
   - Intuition: eigenvalues and eigenvectors are continuous functions of matrix entries

Symmetric Positive Definite and Semidefinite Matrices

1. Geometry of quadratic forms
   - Conditions on eigenvalues for positive (semi)definite matrices
   - Example: finding equiprobable contours for the two dimensional Gaussian distribution with inverse covariance

$$\Lambda_x^{-1} = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$$

2. Matrix square roots
   - Definition and construction from diagonalization
   - Nonuniqueness and the unique symmetric, positive semidefinite square root
3. Applications of square roots
   - Simulation (shaping)
   - Decorrelation (whitening)
4. Principal components analysis (PCA)
   - Definition and relation to eigendecomposition
   - Direct calculation from empirical data via the singular value decomposition (SVD)