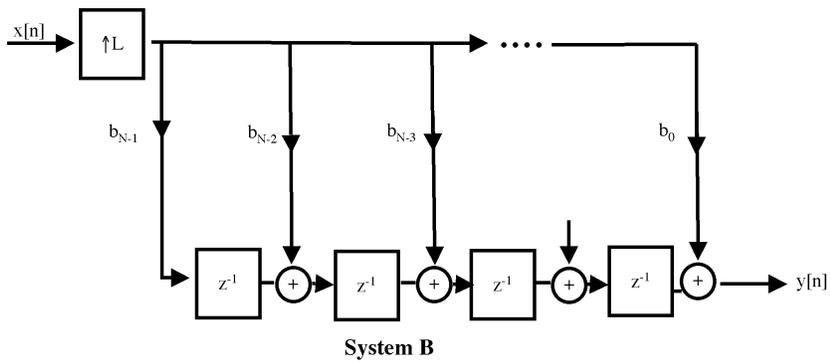
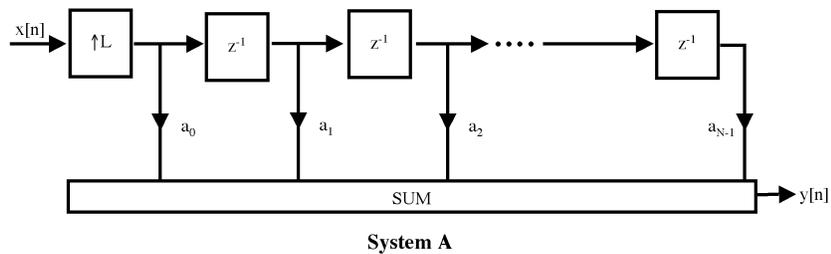


Lecture 11
Multirate Systems and Polyphase Structures

Reading: Section 4.7 in Oppenheim, Schafer & Buck (OSB).

Consider the two systems depicted below.

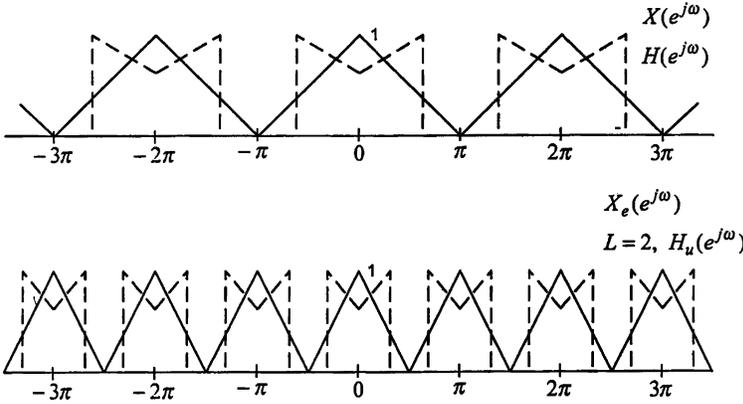


The following questions can be posed:

- (1) How would you choose b_0, \dots, b_{N-1} so that System A and System B are identical?
- (2) How would you make System A more efficient?
- (3) How would you make System B more efficient?

While careful inspection shows that $b_k = a_k$ is the answer to (1), meaningful responses to (2) and (3) are best given after first posing the comeback, “What do we mean by ‘efficient?’” Certainly distributing the compressor in System B to the other side of the b_k multipliers gives a system which is more efficient in terms of the number of multiplications per unit time, but this manipulation also makes the system less efficient in terms of the number of expander blocks. Comparing these systems to the ones at the beginning of the previous lecture will help extend already-developed intuition about implementing efficient (in whatever sense) downsampling systems to the upsampling case. The goal of this lecture is to further develop and solidify that intuition.

Recall the downsampling noble identity from OSB Subsection 4.7.1, depicted in OSB Figure 4.30. There also exists a closely-related identity for upsampling systems which is discussed in that subsection and which is depicted in OSB Figure 4.31. Further insight about the upsampling identity can be gained by considering its behavior in the frequency domain:



The top subfigure shows two DTFTs, a signal $X(e^{j\omega})$ and filter $H(e^{j\omega})$. These respectively correspond to $x[n]$ and $H(z)$ in OSB Figure 4.31(a). The bottom subfigure illustrates the effective input signal DTFT and filter DTFT after each being processed by an expander-by-2 block. It should therefore stand to reason that a similar bottom subfigure could also be generated by first expanding the input signal and then filtering it by an expanded version of the original filter, as in OSB Figure 4.31(b). (Although this argument is by no means a rigorous proof of the upsampling identity, it should help develop intuition about its behavior.)

As in the previous lecture, the question of how this identity might be used to design efficient multirate systems is now posed. Recall from the previous lecture that when a compressor-by- M follows a discrete-time filter $h[n]$ to make a downsampling system as

$$x[n] \longrightarrow \boxed{h[n]} \longrightarrow \boxed{\downarrow M} \longrightarrow y[n],$$

we can move all of the multipliers in the filter to the low-rate (right) side of the compressor block. This is done by running the filter's impulse response $h[n]$ through the identity system in OSB Figure 4.33, picking off the polyphase components $e_k[n]$, and forming an equivalent realization of the filter $h[n]$ as in OSB Figure 4.34. When this new structure is used to implement the filter $h[n]$ in our downsampling system, a series of flow graph manipulations and applications of the downsampling noble identity give an equivalent system which requires fewer multiplications per unit time than a corresponding direct-form, feed-forward implementation.

Extending this intuition to the upsampling case, interpolation systems arranged as in OSB Figure 4.38 can be similarly decomposed and manipulated so that all LTI blocks are on the low-rate (left) side of the expander. This is done by first decomposing the filter $h[n]$ into its polyphase components using again the identity system in OSB Figure 4.33 (and letting $M = L$ there). Our newly-acquired polyphase components $e_k[n]$ are then used to make the equivalent interpolation system depicted in OSB Figure 4.39, and the system in OSB Figure 4.40 is finally arrived at by applying the upsampling noble identity. Note that in this system, which is equivalent to the one in OSB Figure 4.38, all LTI blocks fall on the low-rate side of the expanders.