

MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Electrical Engineering and Computer Science

6.341 DISCRETE-TIME SIGNAL PROCESSING
Fall 2004

MIDTERM EXAM

Tuesday, November 9, 2004

- This is a closed book exam, but two $8\frac{1}{2}'' \times 11''$ sheets of notes (both sides) are allowed.
- Calculators are not allowed.
- Make sure you have all 14 numbered pages of this exam.
- There are 4 problems on the exam.
- The problems are not in order of difficulty. We recommend that you read through all the problems, then do the problems in whatever order suits you best. **Note, too, that very often the later parts of a problem can be done independently of earlier parts.**
- A correct answer does not guarantee full credit, and a wrong answer does not guarantee loss of credit. You should clearly but concisely indicate your reasoning and **show all relevant work**. Your grade on each problem will be based on our assessment of your level of understanding as reflected by what you have written in the space provided.
- Please be neat—we can't grade what we can't decipher.
- Only the booklet supplied is to be handed in. **No additional pages will be considered in the grading.** You may want to work things through on the scratch paper provided first and then neatly transfer the work you would like us to look at into the exam booklet. Let us know if you need additional scratch paper.
- We will be following a grading strategy which focuses on your level of understanding of the material associated with each problem. Specifically, when we grade each part of a problem we will do our best to assess, from your work, your level of understanding.

Based on our judgement, each part will receive one of the following: E (EXCELLENT UNDERSTANDING), G (GOOD), R (REASONABLE), M (MARGINAL) or U (UNSATISFACTORY). Where appropriate, we may also use grades of E/G, G/R, R/M, or M/U so that there is a finer scale.

On each part of each exam question we will also indicate the percentage of the total exam grade represented by that part, and your numerical score on the exam will then be calculated accordingly. (For example if a part represents 16% of the exam grade, then E=16, E/G=14, G=12, G/R=10, R=8, R/M=6, M=4, M/U=2, U=0.)

This system (which we've used in the course many times in the past) is designed to focus your attention and ours on your level of understanding of the material, rather than on what calculations and miscalculations receive or lose points. Consequently, in looking at your graded exam, we ask that you focus on our judgement of your level of understanding. If you honestly feel that we've misjudged it based on what you've written in the exam booklet, then we'd be happy to discuss it with you.

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NAME: _____

| Problem | Grade | Points | Grader |
|---------|-------|--------|--------|
| 1 (a) | | | |
| 1 (b) | | | |
| 1 (c) | | | |
| 1 (d) | | | |
| 2 (a) | | | |
| 2 (b) | | | |
| 2 (c) | | | |
| 3 (a) | | | |
| 3 (b) | | | |
| 4 (a) | | | |
| 4 (b) | | | |
| 4 (c) | | | |
| Total | | | |

Problem 1 (28%)

In figure 1-1 are two systems consisting of a compressor and an expander.

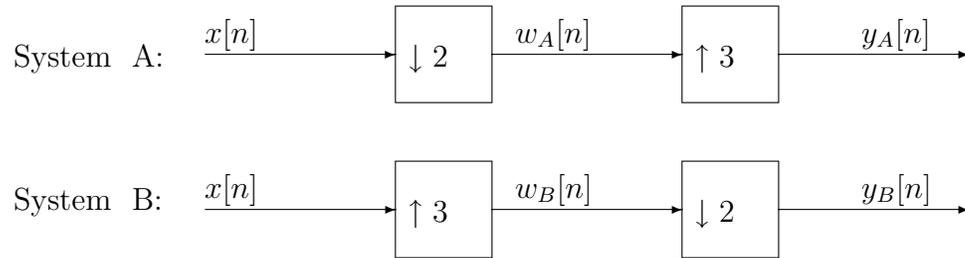


Figure 1-1:

- (a) (4%) For $x[n]$ as shown in figure 1-2 sketch $y_A[n]$ and $y_B[n]$ (assume $x[n] = 0$ outside the interval shown).

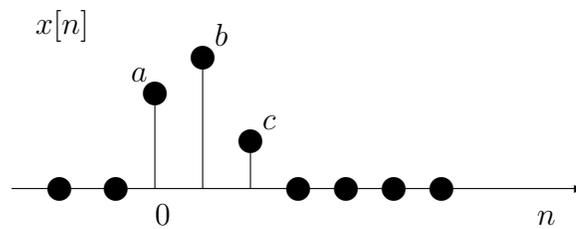


Figure 1-2:

Name: _____

(b) (6%) For $X(e^{j\omega})$ as shown in figure 1-3, sketch $W_a(e^{j\omega})$ and $Y_A(e^{j\omega})$.

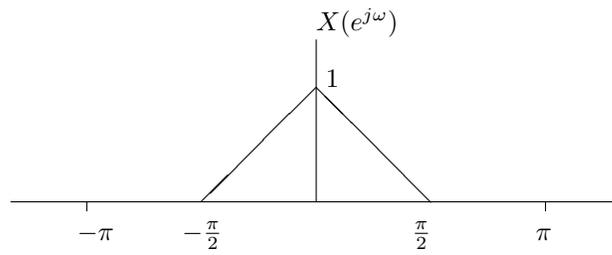


Figure 1-3:

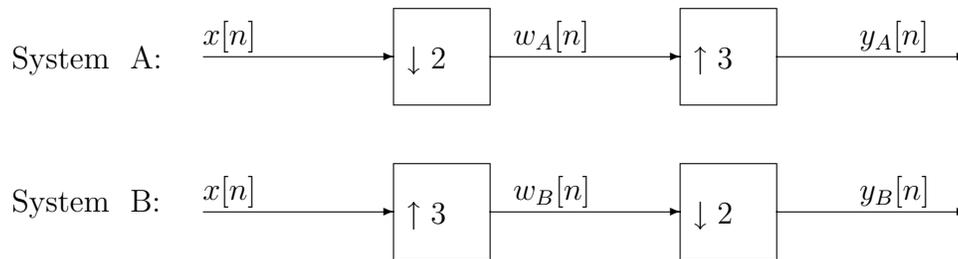


Figure 1-1 repeated for your convenience.

- (c) (8%) $X(e^{j\omega})$ denotes the Fourier transform for an arbitrary $x[n]$. Express $Y_B(e^{j\omega})$ in terms of $X(e^{j\omega})$. Your answer should be in the form of an equation, not a sketch for a specific Fourier transform.

- (d) (10%) For any arbitrary $x[n]$, will $y_A[n] = y_B[n]$? If your answer is yes, algebraically justify your answer. If your answer is no, clearly explain or give a counterexample.

Problem 2 (28%)

Consider the system in Figure 2-1.

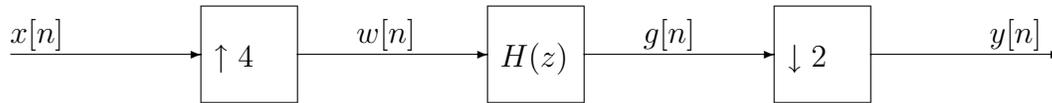


Figure 2-1:

We would like to implement the overall system (i.e. from $x[n]$ to $y[n]$) as efficiently as possible in terms of the number of multiplies per output sample, the total number of compressor/expander blocks, and the total number of delay elements.

- (a) (8%) If $H(z) = \frac{1 - cz^{-1}}{1 - az^{-1}}$, where a and c are arbitrary and the system must be implemented exactly as shown in figure 2-1, with $H(z)$ in direct form, determine the required number of multiplies for each value of $y[n]$.

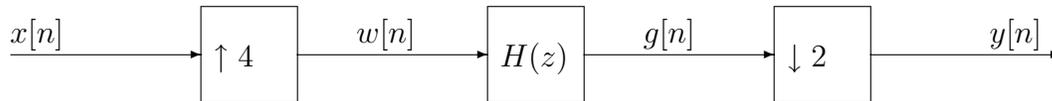


Figure 2-1 repeated for your convenience

For parts (b) and (c) assume that the cost of each multiply per output sample $y[n]$ is 10, the cost of each compressor or expander block is 5, and the cost of each delay is 2.

- (b) (10%) If $H(z) = \frac{1 - cz^{-1}}{1 - az^{-1}}$, determine the flowgraph or block diagram of the overall system which minimizes the implementation cost of the overall system in Figure 2-1, and the total cost of the implementation.

Total Cost:

Name: _____

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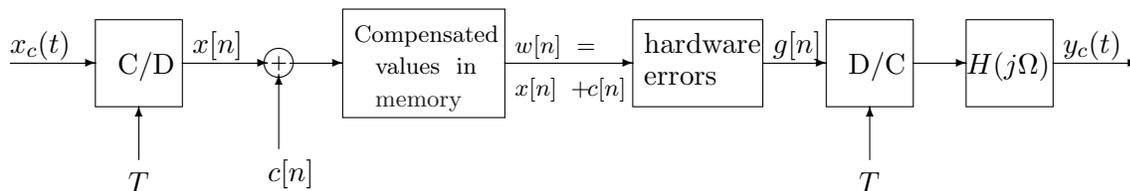
For parts (b) and (c) assume that the cost of each multiply per output sample $y[n]$ is 10, the cost of each compressor or expander block is 5, and the cost of each delay is 2.

- (c) (10%) If $H(z) = \frac{1}{1 - az^{-4} - bz^{-8}}$, determine the flowgraph or block diagram of the overall system which minimizes the implementation cost of the overall system in Figure 2-1, and the total cost of the implementation.

Total Cost:

Problem 3 (20%)

A bandlimited signal $x_c(t)$ has been sampled and the samples have been stored in memory. The samples are to be converted to a continuous-time signal $y_c(t)$ through an ideal C/D converter. It is known that because of a hardware fault, the input to the D/C will be faulty. The objective in this problem is to consider the possibility of pre-compensating $x[n]$ in memory so that the reconstructed signal $y_c(t)$ at the output will equal $x_c(t)$



$$\begin{aligned}
 X_c(j\Omega) &= 0, \text{ for } |\Omega| > \frac{2\pi}{3T} \\
 w[n] &= x[n] + c[n] \\
 X(e^{j\omega}) &= 0, \text{ for } \frac{2\pi}{3} < |\omega| < \pi \\
 H(j\Omega) &= \begin{cases} 1 & \text{for } |\Omega| \leq \frac{2\pi}{3T} \\ 0 & \text{otherwise} \end{cases}
 \end{aligned}$$

- (a) (10%) For this part assume that $c[n] = 0$ (i.e. there's no compensation and, consequently, $w[n] = x[n]$.) Assume further that the hardware errors have the effect of adding an undesired signal to $w[n]$, i.e. $g[n] = x[n] + e[n]$. Under what constraints on $e[n]$ or its Fourier transform $E(e^{j\omega})$ will $y_c(t) = x_c(t)$? Clearly explain your answer.

- (b) (10%) For this part assume that the effect of the hardware error is to always force $g[0]$ to be zero. In designing the compensating signal we note that if $c[0] = -x[0]$ then $w[0] = 0$. Then the hardware error will not affect $w[n]$, i.e. $g[n] = w[n]$. With this in mind and considering your answer in (a), determine an appropriate choice for the compensating signal $c[n]$ so that $y_c(t) = x_c(t)$.

Problem 4 (24%)

For this problem you may find useful the lecture slides which we have reproduced on page 14.

Consider the signal

$$s[n] = \alpha \left(\frac{2}{3}\right)^n u[n] + \beta \left(\frac{1}{4}\right)^n u[n]$$

where α and β are constants.

We wish to linearly predict $s[n]$ from its past p values using the relationship

$$\hat{s}[n] = \sum_{k=1}^p a_k s[n-k]$$

where the coefficients a_k are constants.

The coefficients a_k are chosen to minimize the prediction error

$$\mathcal{E} = \sum_{n=-\infty}^{+\infty} (s[n] - \hat{s}[n])^2$$

- (a) (4%) With $\phi_s[m]$ denoting the autocorrelation of $s[n]$, write the equations for the case $p = 2$ the solution to which will result in a_1, a_2 (you do not need to derive the equations).

(b) (10%) Determine a pair of values for α and β such that when $p = 2$, the solution to the normal equations is $a_1 = \frac{11}{12}$ and $a_2 = -\frac{1}{6}$. Is your answer unique? Explain.

(c) (10%) If $\alpha = 8$ and $\beta = -3$, determine the reflection coefficient k_3 , resulting from using the Levinson recursion to solve the normal equations for $p = 3$. Is that different from k_3 when solving for $p = 4$? (An answer with no explanation will receive no credit.)

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Levinson-Durbin Recursion

$$\sum_{k=1}^p a_k \phi_s[i-k] = \phi_s[i] \quad i = 1, 2, \dots, p$$

$$T_p = \begin{bmatrix} \phi_s[0] & \phi_s[1] & \cdots & \phi_s[p-1] \\ \phi_s[1] & \phi_s[0] & \cdots & \\ \vdots & & & \\ \phi_s[p-1] & \cdots & & \phi_s[0] \end{bmatrix}$$

$$\alpha_p = [a_1, a_2, \dots, a_p]^T \quad r_p = [\phi_s[1] \phi_s[2] \dots \phi_s[p]]^T$$

$$\beta_p = [a_p, a_{p-1}, \dots, a_1]^T \quad \rho_p = [\phi_s[p] \phi_s[p-1] \dots \phi_s[1]]^T$$

$$k_{p+1} = \frac{\phi_s[p+1] - (\rho_p)^T \alpha_p}{\phi_s[0] - (r_p)^T \alpha_p}$$

$$\varepsilon_p = -k_{p+1} \beta_p$$

$$\alpha_{p+1} = \begin{bmatrix} \alpha_p \\ 0 \end{bmatrix} + \begin{bmatrix} \varepsilon_p \\ k_{p+1} \end{bmatrix} = \begin{bmatrix} a_1^{(p)} \\ a_2^{(p)} \\ \vdots \\ a_p^{(p)} \\ 0 \end{bmatrix} - k_{p+1} \begin{bmatrix} a_p^{(p)} \\ a_{p-1}^{(p)} \\ \vdots \\ a_1^{(p)} \\ -1 \end{bmatrix} \quad (7)$$

$a_k^{(p)}$ is the a_k th coefficient for the p th order filter.