

**Problem Set 7**

**The Discrete Fourier Transform, Linear Filtering with the DFT**

**Issued:** Tuesday, October 25, 2005.

**Due:** Tuesday, November 1, 2005.

---

**Reading:** Sections 8.1 - 8.7 in the text.

**Reminders:** Project I is due in class on November 1.

The midterm exam will be held on November 8 from 11:00 AM to 1:00 PM.

It will cover material in Problem Sets 1 through 7. The exam is closed book. However, you may bring two  $8\frac{1}{2}'' \times 11''$  sheets of handwritten notes (4 sides total).

---

**Problem 7.1**

Consider the signal

$$s[n] = \left(\frac{1}{3}\right)^n u[n] + \left(-\frac{1}{2}\right)^n u[n].$$

We wish to model this signal using second-order ( $p = 2$ ) linear prediction.

- (a) Write the normal equations for  $p = 2$  to obtain equations for  $a_1, a_2$  in terms of  $\phi_s[m]$ .
- (b) Find the values of  $\phi_s[0]$ ,  $\phi_s[1]$ , and  $\phi_s[2]$  for the signal  $s[n]$  given above.
- (c) Solve the system of equations from part (a) using the values you found in part (b) to get values for the  $a_k$ 's. You may find MATLAB or another computer tool useful to solve these equations.
- (d) Suppose you wish to model the signal now with  $p = 3$ . Write the normal equations for this case.
- (e) Find the value of  $\phi_s[3]$ .
- (f) Solve for the values of  $a_k$  when  $p = 3$ . You may find MATLAB or another computer tool useful to solve these equations.
- (g) Would the values of  $a_1, a_2$  you found in (f) change if we model the signal with  $p = 4$ ?
- (h) You should have found a fundamental difference between the results from this problem and the results from Problem 6.7 on the last problem set. What is the fundamental difference between the examples in the two problems?
- (i) Determine a set of coefficients for which  $s[n]$  for  $n \geq 2$  is exactly linearly predictable from  $s[n - 1]$  and  $s[n - 2]$ . **Hint:** write down the difference equation for which the impulse response is  $s[n]$  and set  $n \geq 2$ .

**Problem 7.2**

OSB Problem 8.31

**Problem 7.3**

OSB Problem 8.32

**Problem 7.4**

OSB Problem 8.37

**Problem 7.5**

OSB Problem 8.46

**Problem 7.6**

OSB Problem 8.59

Note: The output relations for modules I-IV apply for all  $n$ .**Problem 7.7**

A discrete-time LTI filter  $S$  is to be implemented using the overlap-save method. Recall that in the overlap-save method the input is divided into *overlapping* blocks, as opposed to the overlap-add method where the input blocks are non-overlapping. For this implementation, the input signal  $x[n]$  will be divided into overlapping 256-point blocks  $x_r[n]$ . Adjacent blocks will overlap by 255 points so that they differ by only one sample. Mathematically, we have the following relation between  $x_r[n]$  and  $x[n]$ , where  $r$  ranges over all integers and we obtain a different block  $x_r[n]$  for each value of  $r$ :

$$x_r[n] = \begin{cases} x[n+r] & 0 \leq n \leq 255 \\ 0 & \text{otherwise} \end{cases}$$

Each block is processed by taking the 256-point DFT of  $x_r[n]$ , multiplying the result with  $H[k]$  given below, and taking the 256-point inverse DFT of the product. The unaliased samples from each block computation (in this case only a single sample per block) are then “saved” as part of the overall output.

$$H[k] = \begin{cases} 1 & 0 \leq k \leq 31 \\ 0 & 32 \leq k \leq 224 \\ 1 & 225 \leq k \leq 255 \end{cases}$$

In this problem you may assume that the overlap-save method has been correctly implemented.

- (a) Is  $S$  an ideal frequency-selective filter? Justify your answer.
- (b) Is the impulse response of  $S$  real valued? Justify your answer.
- (c) Determine the impulse response of  $S$ .