

**Problem Set 2**

**FIR/IIR systems, Recursive and Non-Recursive systems, Interpretation of phase, Linear phase, Minimum phase and all-pass systems. Review of DT Processing of CT Signals, Sample-rate conversion.**

**Issued:** Tuesday, September 13, 2005.

**Due:** Tuesday, September 20, 2005.

---

**Reading:** Review the concept of group delay in section 5.1.2 and 5.3. Section 5.4 discusses the relationship between frequency response magnitude and phase. All-pass systems are discussed in section 5.5 and minimum-phase systems are discussed in 5.6. Review OSB chapter 4 section 4.0 - 4.5. You should carefully read 4.6. **Note:** The Background Exam is scheduled for Thursday, September 22 from 11am to 1pm. Questions on the Background Exam may involve aspects of material covered in lecture through September 15

(Lecture 3), up to but not including sample-rate conversion. As indicated in the information handout, the exam will be graded, but the grade will not count in your final grade. However, a grade below 80 will indicate important weakness in your background.

---

**Problem 2.1**

Many properties of a discrete-time sequence  $h[n]$  or an LTI system with impulse response  $h[n]$  can be discerned from a pole-zero plot of  $H(z)$ . **In this problem we are concerned only with causal  $h[n]$ .** Clearly describe the  $z$ -plane characteristic that corresponds to each of the following properties:

- (i) Real-valued impulse response:
- (ii) Finite impulse response:
- (iii)  $h[n] = h[2\alpha - n]$  where  $2\alpha$  is an integer:
- (iv) Minimum phase:
- (v) All-pass:

**Problem 2.2**

For all parts of this problem  $H(e^{j\omega})$  is the frequency response of a DT filter and can be expressed in polar coordinates as

$$H(e^{j\omega}) = A(\omega)e^{j\theta(\omega)}$$

where  $A(\omega)$  is even and real-valued and  $\theta(\omega)$  is a continuous, odd function of  $\omega$  for  $-\pi < \omega < \pi$ , *i.e.*,  $\theta(\omega)$  is what we have referred to as the *unwrapped phase*. Recall:

- The *group delay*  $\tau(\omega)$  associated with the filter is defined as

$$\tau(\omega) = -\frac{d\theta(\omega)}{d\omega} \quad \text{for } |\omega| < \pi.$$

- An LTI filter is called *minimum phase* if it is stable and causal and has a stable and causal inverse.

For each of the following statements, state whether it is **TRUE** or **FALSE**. If you state that it is **TRUE**, give a clear, brief justification. If you state that it is **FALSE**, give a simple counterexample with a clear, brief explanation of why it is a counterexample. **No credit will be given for true/false answers without explanations.**

- “If the filter is causal, its group delay must be nonnegative at all frequencies in the range  $|\omega| < \pi$ .”
- “If the group delay of the filter is a positive constant integer for  $|\omega| < \pi$  the filter must be a simple integer delay.”
- “If the filter is minimum phase and all the poles and zeros are on the real axis then  $\int_0^\pi \tau(\omega)d\omega = 0$ .”

### Problem 2.3

$H(z)$  is the system function for a stable LTI system and is given by:

$$H(z) = \frac{(1 - 2z^{-1})(1 - 0.75z^{-1})}{z^{-1}(1 - 0.5z^{-1})}.$$

- $H(z)$  can be represented as a cascade of a minimum phase system  $H_1(z)$  and a unity-gain all-pass system  $H_A(z)$ , *i.e.*

$$H(z) = H_1(z)H_A(z).$$

Determine a choice for  $H_1(z)$  and  $H_A(z)$  and specify whether or not they are unique up to a scale factor.

- $H(z)$  can be expressed as a cascade of a minimum-phase system  $H_2(z)$  and a generalized linear phase FIR system  $H_L(z)$ :

$$H(z) = H_2(z)H_L(z).$$

Determine a choice for  $H_2(z)$  and  $H_L(z)$  and specify whether or not these are unique up to a scale factor.

### Problem 2.4

OSB Problem 5.64

**Problem 2.5**

OSB Problem 5.66

**Problem 2.6**

OSB Problem 4.24

**Problem 2.7**

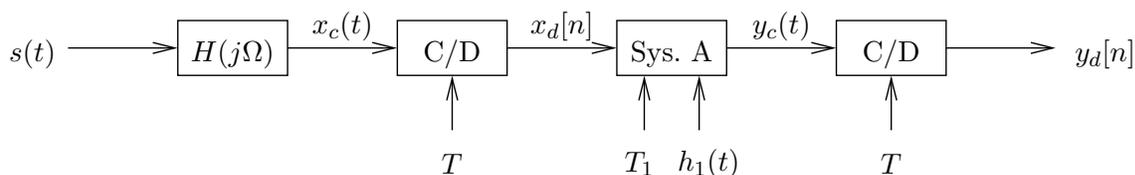
OSB Problem 4.37.

Note: The sampling period  $T$  should be  $\frac{1}{6} \times 10^{-3}$  rather than  $\frac{1}{6} \times 10^{-4}$  in Figure P4.37-2**Problem 2.8**

OSB Problem 4.40

**Problem 2.9**

In the system shown below, the individual blocks are defined as indicated.



$$H(j\Omega): \quad H(j\Omega) = \begin{cases} 1, & |\Omega| < \pi \cdot 10^{-3} \text{ rad/sec} \\ 0, & |\Omega| > \pi \cdot 10^{-3} \text{ rad/sec} \end{cases}$$

$$\text{First C/D: } x_d[n] = x_c(nT)$$

$$\text{System A: } y_c(t) = \sum_{k=-\infty}^{\infty} x_d[k] h_1(t - kT_1)$$

$$\text{Second C/D: } y_d[n] = y_c(nT)$$

- Specify a choice for  $T$ ,  $T_1$  and  $h_1(t)$  so that  $y_c(t)$  and  $x_c(t)$  are guaranteed to be equal for any choice of  $s(t)$ .
- State whether your choice in (a) is unique or whether there are other choices for  $T$ ,  $T_1$  and  $h_1(t)$  that will guarantee that  $y_c(t)$  and  $x_c(t)$  are equal. As usual, clearly show your reasoning.
- For this part, we are interested in what is often referred to as *consistent resampling*. Specifically, the system A constructs a continuous-time signal  $y_c(t)$  from  $x_d[n]$  the sequence of samples of  $x_c(t)$  and is then resampled to obtain  $y_d[n]$ . The resampling is referred to as consistent if  $y_d[n] = x_d[n]$ . Determine the most general conditions you can on  $T$ ,  $T_1$  and  $h_1(t)$  so that  $y_d[n] = x_d[n]$ .