

Massachusetts Institute of Technology
Department of Electrical Engineering and Computer Science

6.341: DISCRETE-TIME SIGNAL PROCESSING

Fall 2005

Problem Set 1
Background Review

Issued: Thursday, September 8, 2005.

Due: Tuesday, September 13, 2005.

Reading: 6.341 assumes that you have previously seen and have fluency with the following from the 6.341 text (OSB): All of chapters 2 and 3, chapter 4 through section 4.5, chapter 5 through section 5.3, Appendix A.

Note: The Background Exam is scheduled for Thursday, September 22 from 11 AM - 1 PM.

As indicated in the information handout, the exam will be graded, but the grade will not count in your final grade. However, a grade below 80 will indicate important weakness in your background.

Problems 1.1 - 1.14 should help you in reviewing your background. The solutions to these problems are posted separately on the course web site, so you can grade yourself on them. Also attached is the background exam from Fall 2004, which you may want to use first to test yourself. Please don't consult the answers until you have finished testing yourself; the answers are also posted on the web site.

We don't expect you to have the time to work all 14 problems plus the background exam (although you're of course welcome to). The 14 problems are a reasonable representation of the background that we're hoping you have or will develop quickly, so perhaps you can concentrate on the ones that look least familiar to you. For this problem set we are not asking you to turn in your solutions to any of the problems. Instead, we ask that you turn in:

1. A complete list of all of the problems 1.1 - 1.14 that you worked independently without first consulting the solutions or answers.
 2. From the list in 1, how many you worked correctly.
 3. Your self-grade on the attached Fall 2004 background exam.
 4. Based on your answers to 2 and 3 and additional background studying that you plan to do, your best prediction of your grade on the Background Exam scheduled for September 22.
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Problem 1.1

OSB Problem 2.1 (a)-(e)

Problem 1.2

OSB Problem 2.6

Problem 1.3

OSB Problem 2.11. Repeat for the signal $x[n] = \sin\left(\frac{7\pi}{4}n\right)$

Problem 1.4

OSB Problem 2.55

Problem 1.5

OSB Problem 3.4

Problem 1.6

OSB Problem 3.9. Also, determine the LCCDE relating the input and output.

Problem 1.7

OSB Problem 3.40

Problem 1.8

OSB Problem 3.46

Problem 1.9

Note that in OSB and 6.341 Ω denotes continuous-time frequency and ω denotes discrete-time frequency.

Figure 1.9-1 shows a continuous-time filter that is implemented using an LTI discrete-time filter with frequency response $H(e^{j\omega})$.

- (a) If the CTFT of $x_c(t)$, namely $X_c(j\Omega)$, is as shown in Figure 1.9-2 and $\omega_c = \frac{\pi}{5}$, sketch and label $X(e^{j\omega})$, $Y(e^{j\omega})$ and $Y_c(j\Omega)$ for each of the following cases:
- (i) $1/T_1 = 1/T_2 = 2 \times 10^4$
 - (ii) $1/T_1 = 4 \times 10^4$, $1/T_2 = 10^4$
 - (iii) $1/T_1 = 10^4$, $1/T_2 = 3 \times 10^4$
- (b) For $1/T_1 = 1/T_2 = 6 \times 10^3$, and for input signals $x_c(t)$ whose spectra are bandlimited to $|\Omega| < 2\pi \times 5 \times 10^3$ (but otherwise unconstrained), what is the maximum choice of the cutoff frequency ω_c of the filter $H(e^{j\omega})$ for which the overall system is LTI? For this maximum choice of ω_c , specify $H_c(j\Omega)$.

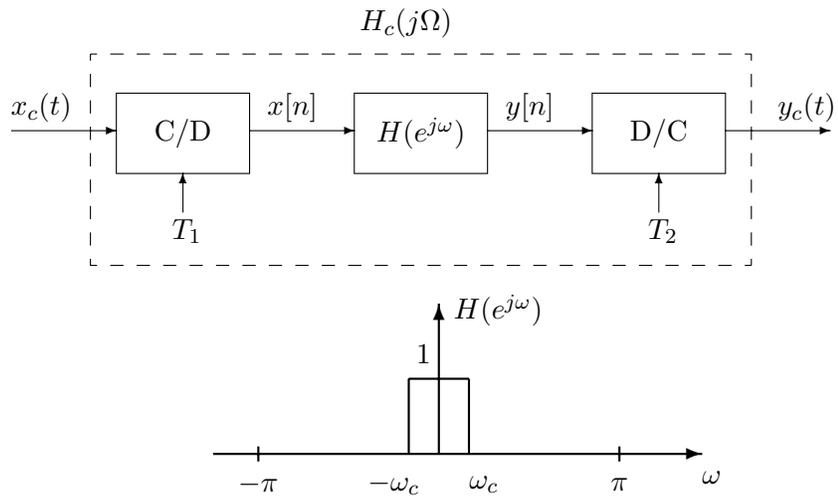


Figure 1.9-1: Problem 1.9.

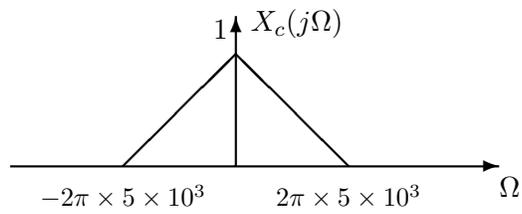


Figure 1.9-2: Problem 1.9, part (a).

Problem 1.10

Filter A is a discrete-time LTI system with input $x[n]$ and output $y[n]$.



The frequency response magnitude and group delay functions for Filter A are shown in Figure 1.10-1. The signal $x[n]$, also shown in Figure 1.10-1, is the sum of three narrowband pulses. Specifically, Figure 1.10-1 contains the following plots:

- $x[n]$.
- $|X(e^{j\omega})|$, the Fourier transform magnitude of $x[n]$.
- Frequency response magnitude plot for filter A.
- Group delay plot for filter A.

In Figure 1.10-2 you are given 4 possible output signals $y_i[n]$, $i = 1, 2, 3, 4$. Determine which one of the possible output signals is the output of filter A when the input is $x[n]$. Provide a justification for your choice.

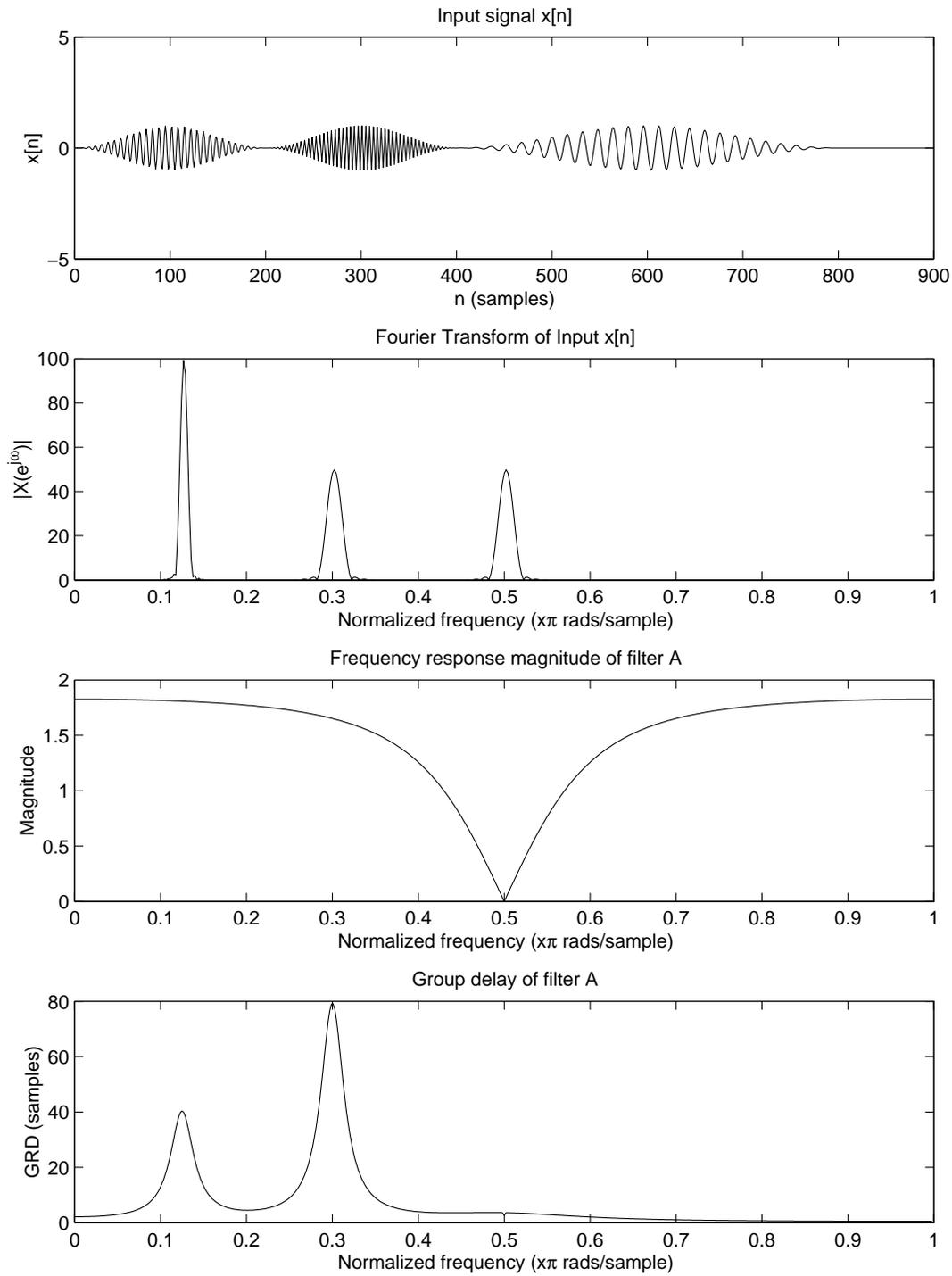


Figure 1.10-1: The signal and the filter for Problem 10

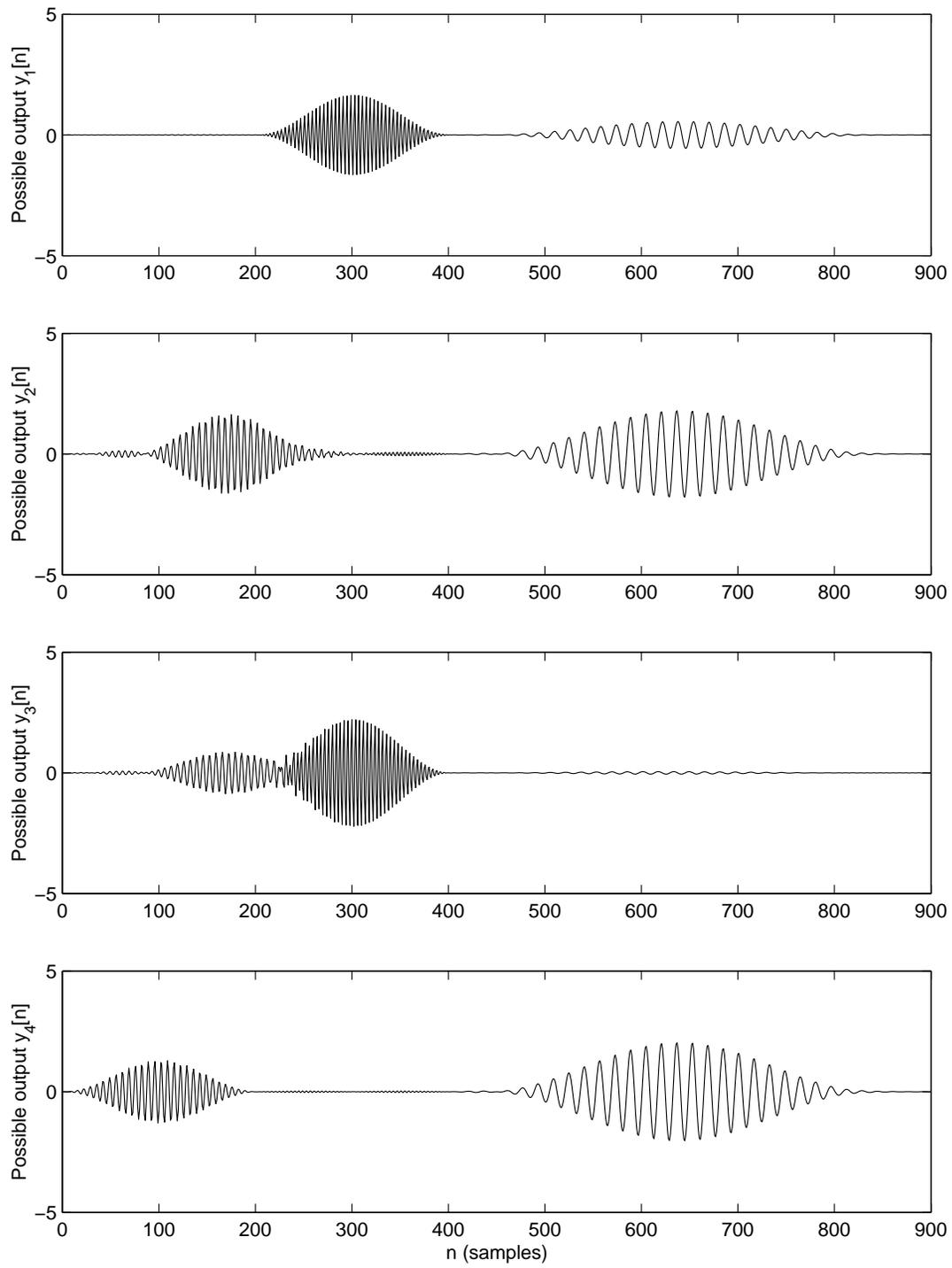


Figure 1.10-2: Possible output signals for Problem 10.

Problem 1.11

Determine the mean and the variance of a random variable with uniform distribution in $[0, \Delta]$

Problem 1.12

$x[n]$ is a zero mean, wide sense stationary random sequence with autocorrelation $R_{xx}[m] = \delta[m]$. $x[n]$ is the input to a LTI system with impulse response

$$h[n] = \begin{cases} 1 & n = 0, 1, 2 \\ 0 & \text{otherwise} \end{cases}$$

The system output is $y[n]$.

(a) Determine $R_{yy}[m]$ and $R_{xy}[m]$ as defined below:

$$\begin{aligned} R_{yy}[m] &= E(y[n+m]y[n]) \\ R_{xy}[m] &= E(x[n+m]y[n]). \end{aligned}$$

(b) Determine the power spectral density of the input and output of the system.

Problem 1.13

OSB Problem 4.5

Problem 1.14

OSB Problem 2.89