

# 6.302 Feedback Systems

Recitation 9: Introduction to the Nyquist Criterion

Prof. Joel L. Dawson

Root locus techniques are good, but they require some detailed information in order for us to use them. All transfer functions involved must be rational, and all of the pole and zero locations must be known. This precludes us from:

- Considering delay:  $e^{-sT}$
- Analyzing stability when only frequency-domain data is available

The Nyquist Stability Criterion is one tool that will help us out. Before we get to that, though, we will need a reminder of how to do conformal mapping with functions of a complex variable.

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## Conformal Mapping

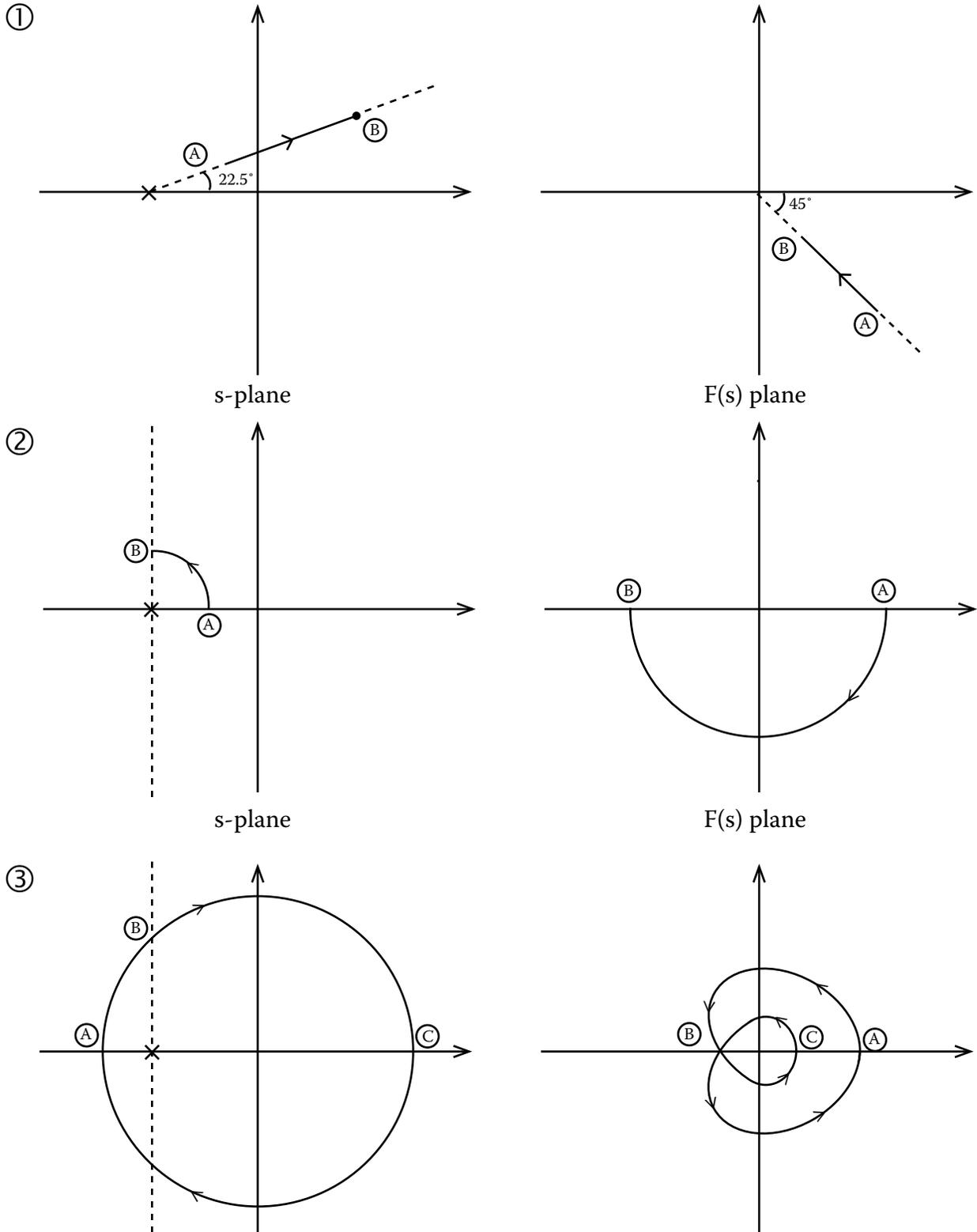
With conformal mapping, we start with a contour in the  $s$ -plane. For every point on that contour, we evaluate the mapping function  $F(s)$ . We then plot the results of these evaluations on a separate set of axes.

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EXAMPLES: Let  $F(s)$  be  $\frac{1}{(s+1)^2}$ . Let's do a mapping for three different contours.



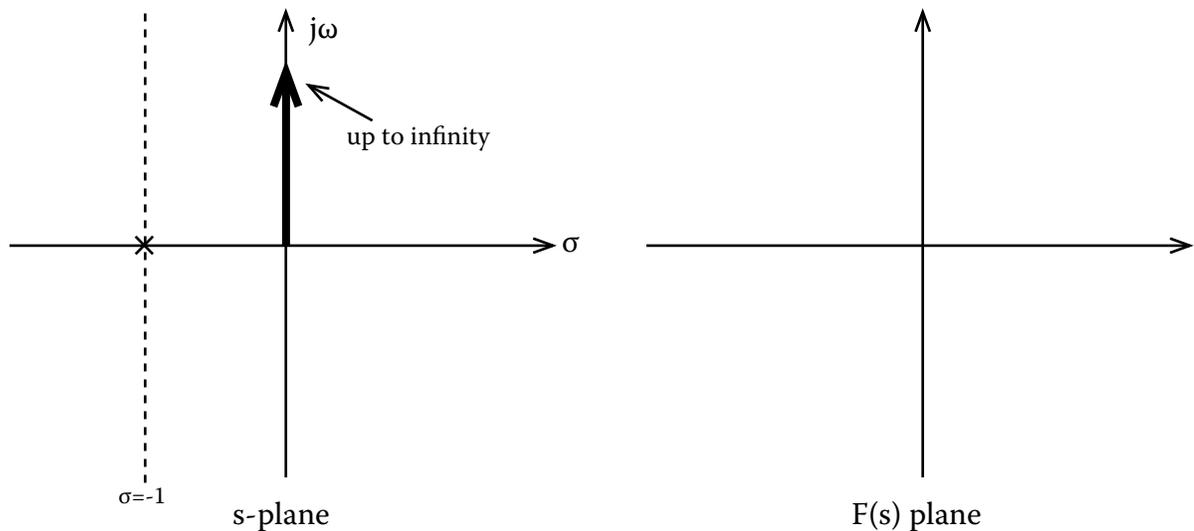
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CLASS EXERCISE

Use the function  $F(s) = \frac{1}{s+1}$  to map the upper half of the  $j\omega$ -axis:



(NOTE: In this case, a bode plot can aid the mapping process.)

Now, we take a principle from complex analysis: Cauchy's Principle of the Argument:

Given a function  $F(s)$  and a closed contour  $C$  in the  $s$ -plane such that  $F(s)$  has no poles or zeros on  $C$ , then

$$N = Z - P$$

Where

- $N = \#$  of positive encirclements of the origin
- $Z = \#$  of zeros of  $F(s)$  inside  $C$
- $P = \#$  of poles of  $F(s)$  inside  $C$

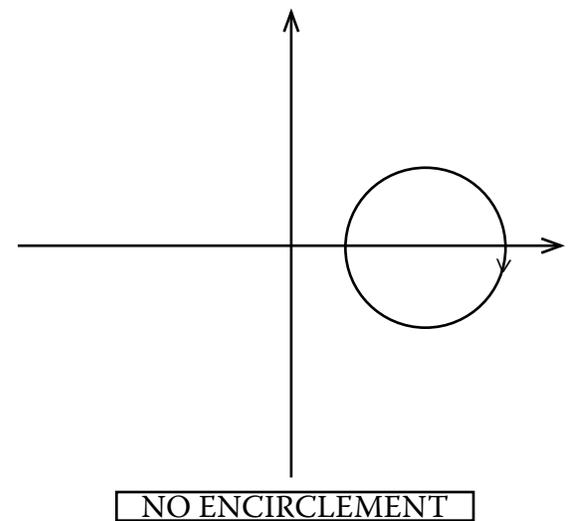
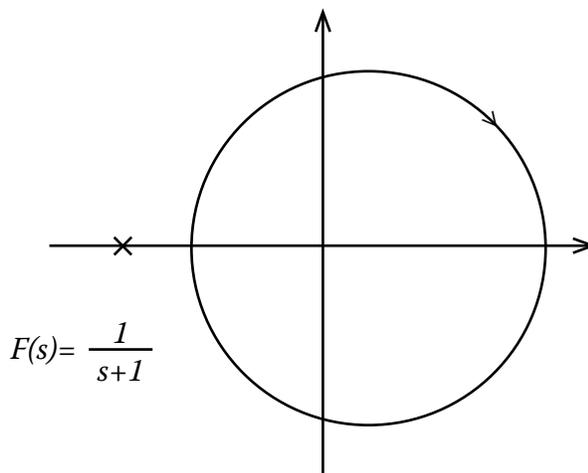
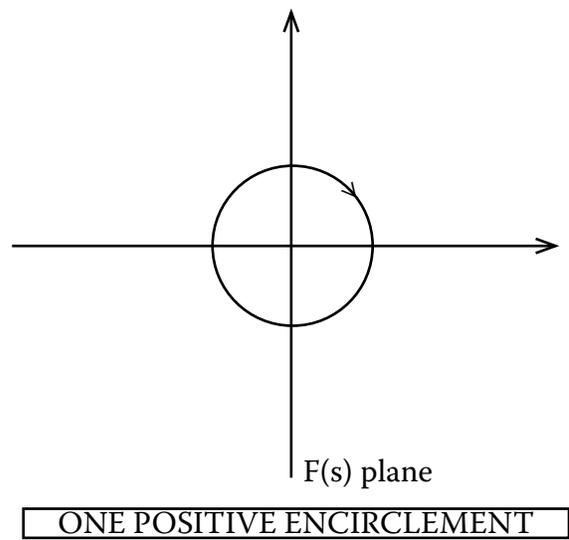
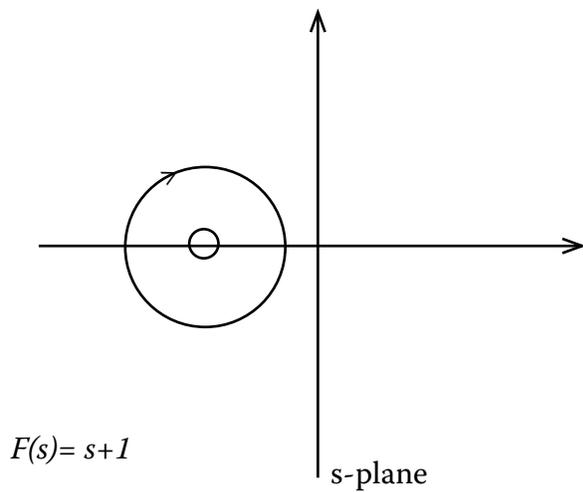
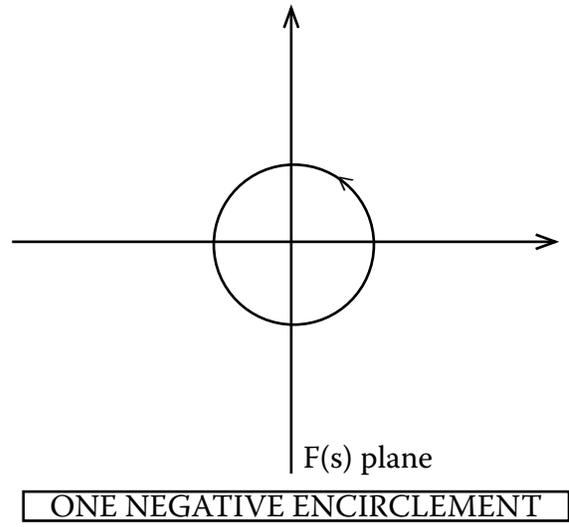
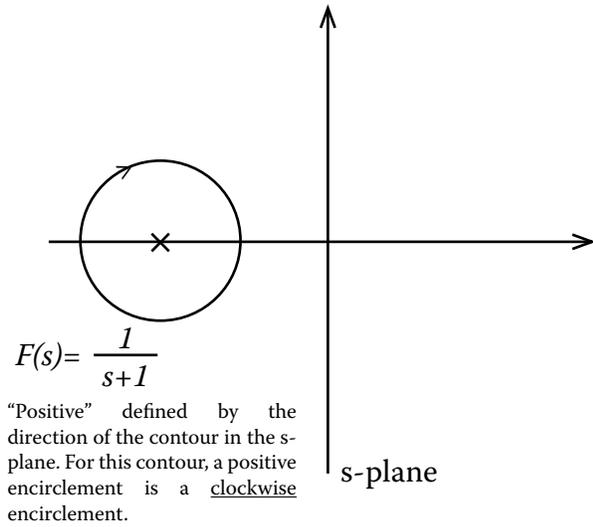
Look back at EXAMPLE 3, and see two negative encirclements.

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EXAMPLES:



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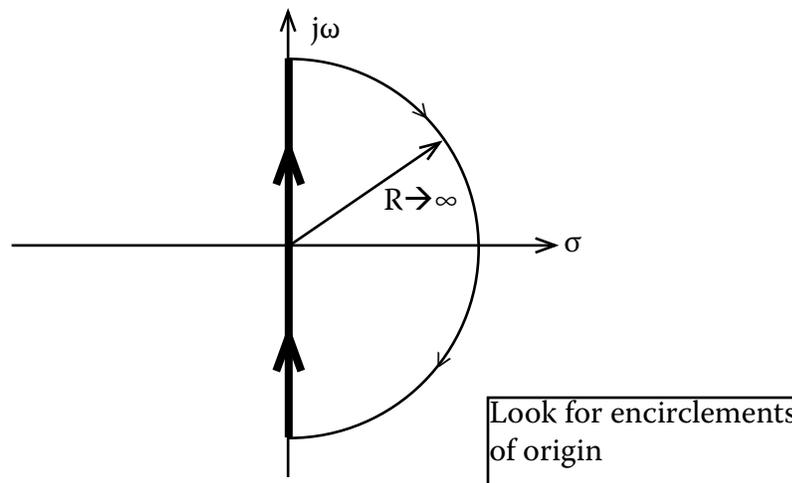
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So...if we have a function  $F(s)$  and we draw a contour around some part of the  $s$ -plane, and if we know how many poles of  $F(s)$  are inside  $C$ , we can deduce the number of zeros that are inside  $C$ .

But what does this have to do with control theory?? We can answer this by answering four other questions:

1. What is a sensible choice for  $F(s)$ ?
2. What is the importance of the zeros of  $F(s)$ ?
3. How do we already know the poles of  $F(s)$ ?
4. Where should we draw our contour  $C$ ?

- ①  $F(s) = 1 + L(s)$
- ② The zeros of  $F(s)$  are the poles of the closed-loop system.
- ③ The poles of  $F(s)$  are the poles of  $L(s)$ .
- ④ The D-contour



# 6.302 Feedback Systems

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So the strategy emerges for finding the closed-loop poles:

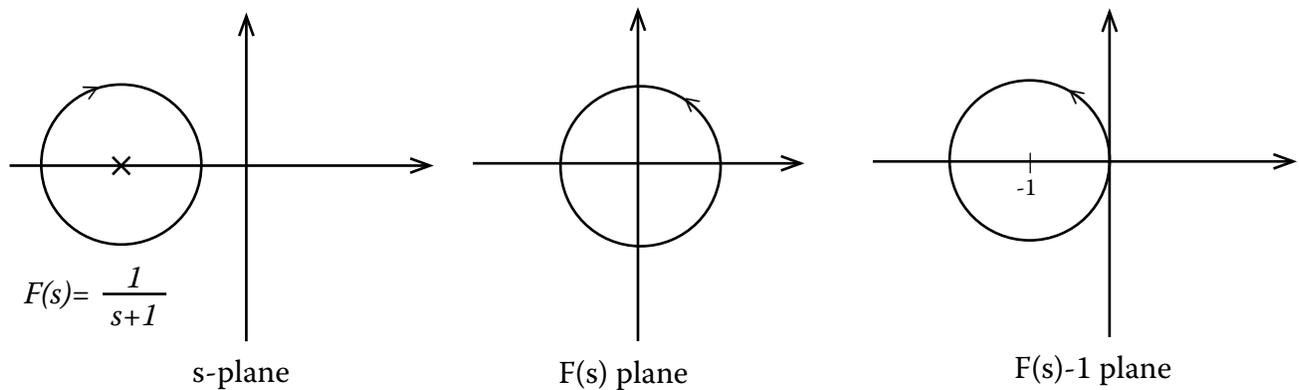
- Plot  $F(s) = 1 + L(s)$  along the D-contour
- Count positive encirclements of the origin in the  $F(s)$  plane

Now, a couple of changes to make our lives easier:

- It would be really nice to be able to work directly with  $L(s)$  rather than  $F(s) = 1 + L(s)$

↪ use a mapping function  $F(s) - 1 = L(s)$

Now it is encirclements of the  $s=-1$  point that matter:



- It would be great not to have to redraw our mapping everytime we change the gain  $k$ .

↪ use  $F(s) = \frac{L(s)}{k} = L_o(s)$

use both of these tricks, and count encirclements of  $-1/k$ .

# 6.302 Feedback Systems

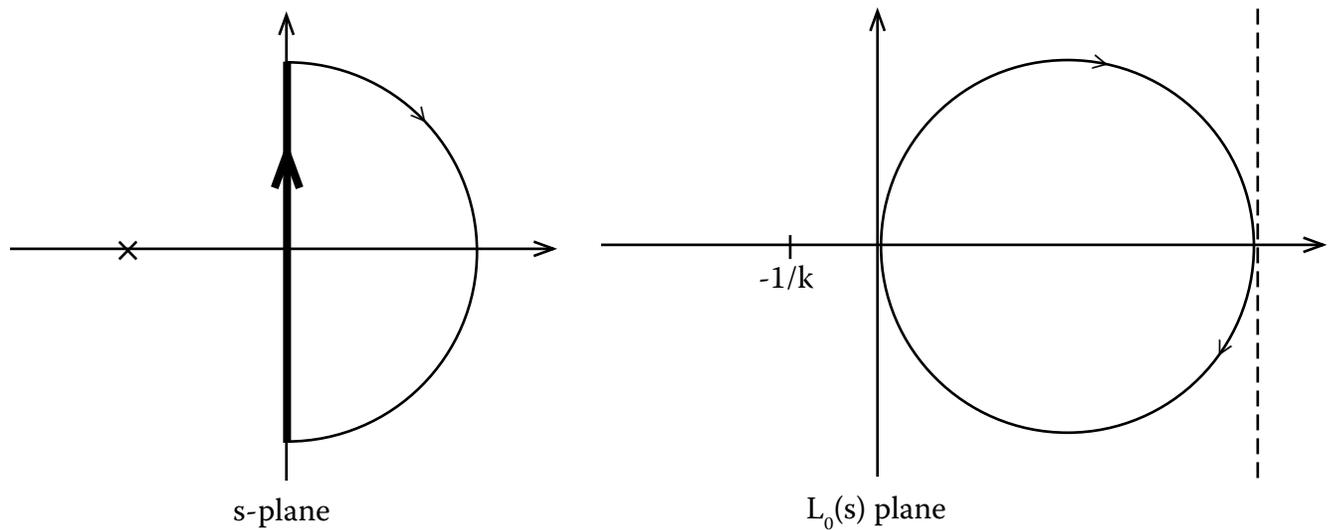
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Finally, then, we arrive at the Nyquist Stability Criterion:

1. Do conformal map of D-contour using  $L_0(s)$  as mapping function.
2. Apply  $Z = N + P$ 
  - $Z = \#$  zeros of  $1 + L(s)$  in RHP
  - $N = \#$  of positive encirclements of  $-1/k$  point
  - $P = \#$  of open-loop poles in RHP.

EXAMPLE:  $L(s) = \frac{k}{s + 1}$



Stability Table:

Range	N	Z	P	Stable?
$0 < k < \infty$	0	0	0	YES
$-\infty < k < -1$	1	1	0	NO
$-1 < k < 0$	0	0	0	YES