6.302 Feedback Systems

Recitation 8: Root Locus, continued

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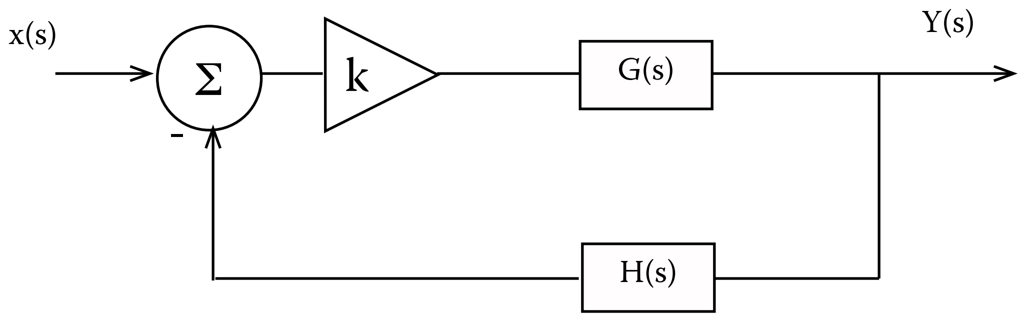
We've been talking about how root locus methods allow us to see how the closed-loop poles move as we vary the loop gain. But what about the closed-loop zeros??



$$\frac{Y(s)}{x(s)} = \frac{kG(s)}{1 + kG(s)H(s)} = \frac{kG(s)}{1 + kL_0(s)}$$

The closed-loop zeros are the zeros of *G(s)* and the poles of *H(s)*. The locations of theses zeros do not very with loop gain *k*. Notice that we do not keep careful track of the closed-loop zero locations with root locus methods.

RULE #5: As *k* gets very large, *P-Z* branches go off to infinity (rule 2). These branches approach asymptotes as angles to the real axis of

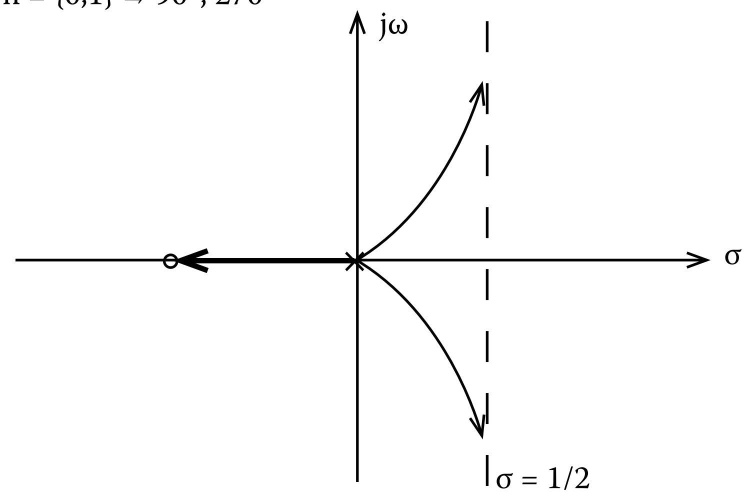
$$\alpha_n = \frac{(2n + 1)\, 180^\circ}{P - Z}$$

Where $n = 0....(P-Z-1)$ and the centroid of these asymptotes is on the real axis at

$$\sigma_a = \frac{\Sigma p_i - \Sigma z_i}{P - Z}$$

EXAMPLE: $L(s) = \frac{k(s+1)}{s^3}$

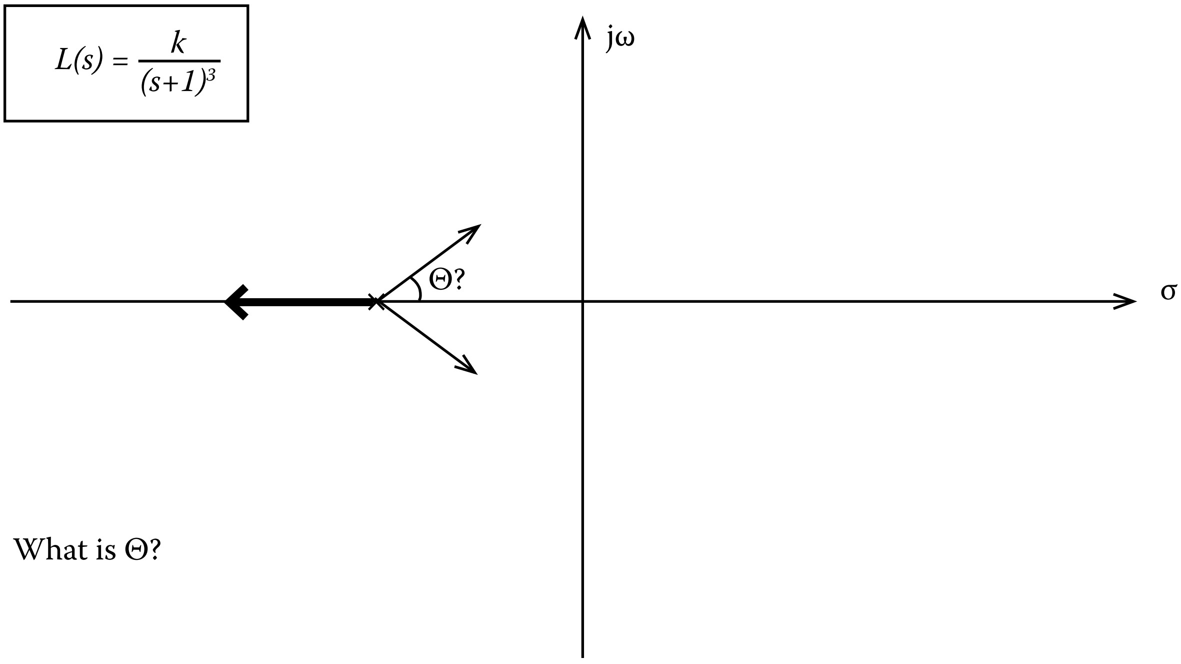
Asymptotes: n = {0,1} → 90˚, 270˚



Centroid: $\frac{0 + 0 + 0 - (-1)}{2} = \frac{1}{2}$

RULE #6: Let's try to derive this one in class, or at least part of it. Suppose that we have a 3rd order pole sitting on the real axis. What are the angles of departure?

$$L(s) = \frac{k}{(s+1)^3}$$



What is Θ?

Rule 6 is a generalizetion of this result. If the mth order pole is to the left of an even number of poles/zeros, the departure angles are

$$\alpha_n = \frac{(2n+1)180^\circ}{m} \qquad n=\{0,1,...m-1\}$$

If to the left of an odd number of singularities,

$$\alpha_n = \frac{2n \cdot 180^\circ}{m}$$

want to introduce one more rule...a complete list can be found in the text.

GRANT's RULE: If there are two or more excess poles than zeros $(P - Z) \geq 2$, then for any gain k, the sum of the real parts of the closed-loop poles is constant.

$$P(s) = (s+a)(s+b)(s+c)(s+d) = 0$$
$$P(s) = s^4 + \omega s^3 + xs^2 + ys + z = 0$$

Coefficient ω is the sum of the roots.

$$P(s) = 1 + L(s) = 1 + k\,\frac{n(s)}{d(s)} = 0$$

$$d(s) + kn(s) = 0$$

for $P\text{-}Z \geq 2$, k does not have an impact on ω.

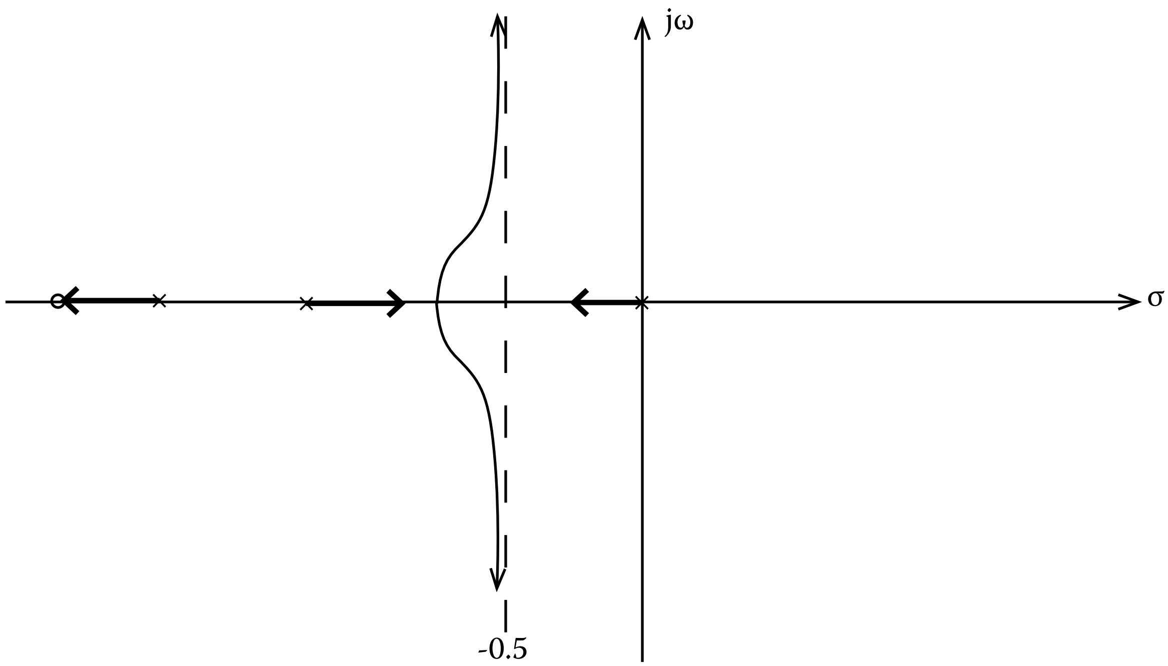
⇒The average distance from the $j\omega$ axis remains constant, EXAMPLE:

$$L(s) = \frac{k(s+4)}{s(s+2)(s+3)}$$

Number of asymptotes: P - Z = 2

Angle of asymptotes: $\frac{(2m+1)\cdot 180^\circ}{2} = 90^\circ, 270^\circ$

Centroid of asymptotes: $\sigma_a = \frac{\Sigma pi - \Sigma zi}{P-Z} = \frac{0 - 2 - 3 + 4}{2} = -0.5$

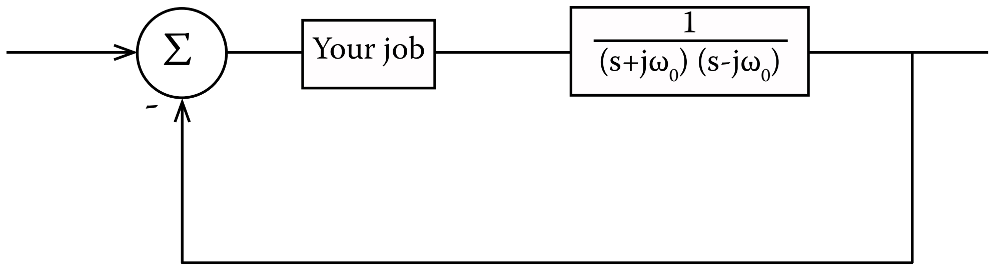


Introduction to Compensation: Making Things Better

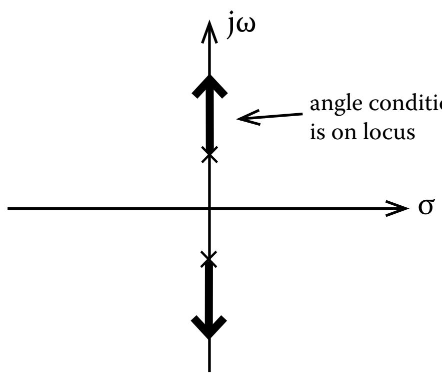
Root locus techniques are one of many tools that we use to design feedback systems to work the way we want them to.

A feedback design problem might go something like this: you're given a "plant," or something you must control; the variable of interest (e.g. velocity, position, etc.), and pecs on performance (maximum overshoot, rise time, bandwidth).

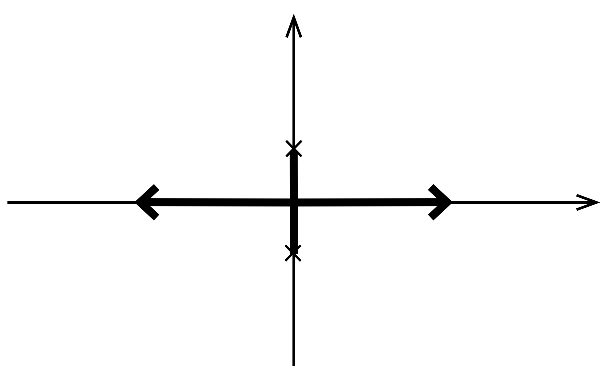
Let's say that you're asked to control a plant that has a complex pair of poles:



If you hadn't already taken 6.302, your first thought might be to put in a gain:

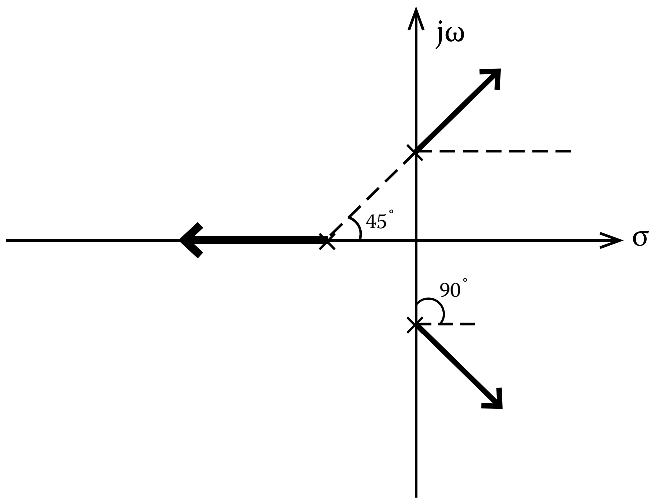


That didn't work: we just get an oscillator higher and higher frequency. What about negative gain (positive feedback??)? New angle condition: $\measuredangle L_0(s) = 0$



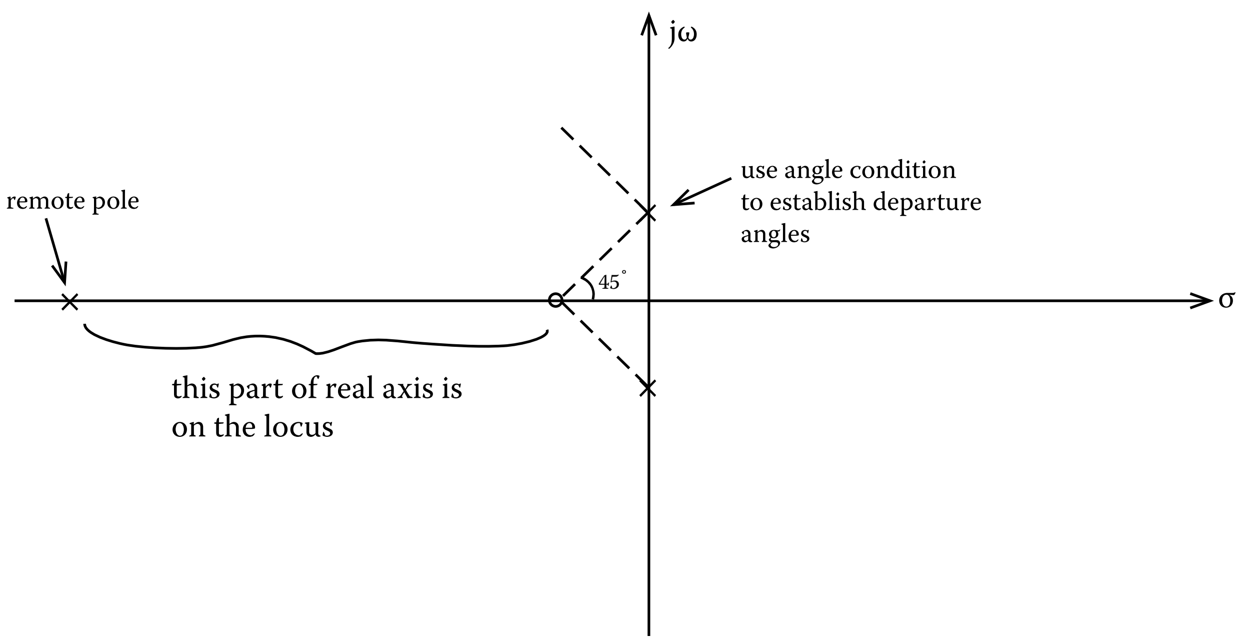
Entire real axis is on locks, as is imaginary axis between poles.

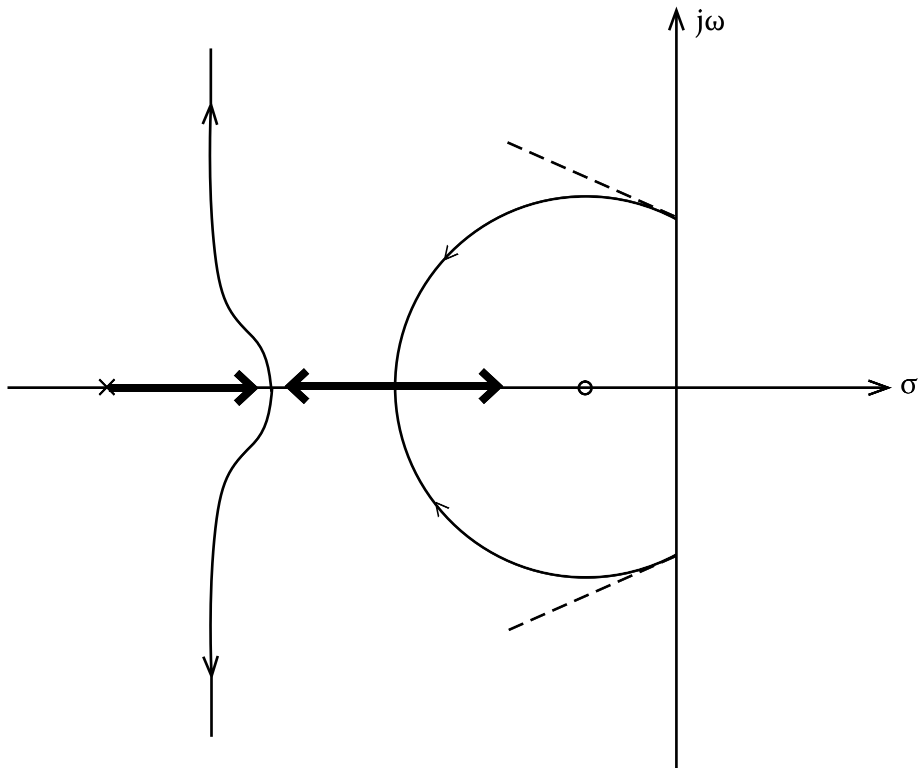
Hmm. Try adding a real pole? (To make the numbers nice, real pole @ $-\omega_0$)



(Notice how we use angle condition to handle conjugate pair of poles).

Running out of options. We can put in a zero, but we know that no physical system can have more zeros than poles. But let's try the next best thing: a zero @ $-\omega_0$ and a <u>remote</u> pole. If the pole is far enough away, we can ignore it in the region near the origin.





So if we pick *k* right, we could wind up with only real closed-loop poles.

Notice that we haven't figured out any quantitative information (just what is *k*, for example?). But root locus helped us to come up with a compensation strategy.