6.302 Feedback Systems

Recitation 3: Block Diagrams (continued)

Prof. Joel L. Dawson

Quick review of Taylor Series Approximation. Suppose we have a complicated function $f(\cdot)$, whose value $f(x)$ we happen to know. If the function $f(\cdot)$ is well-behaved, we can write the equality (not merely approximation):

$$f(x + \Delta x) = f(x) + \frac{1}{1!}\frac{df}{dx}\Delta x + \frac{1}{2!}\frac{d^2 f}{dx^2}(\Delta x)^2 + ...$$

$$= f(x) + \sum_{k=1}^{\infty} \frac{1}{k!}\frac{d^k}{dx^k}(\Delta x)^k$$

When we linearize, we approximate this equality as:

$$f(x + \Delta x) = f(x) + \Delta f \approx f(x) + \frac{df}{dx}\Delta x$$

So we indentify Δf as $\frac{df}{dx}\Delta x$

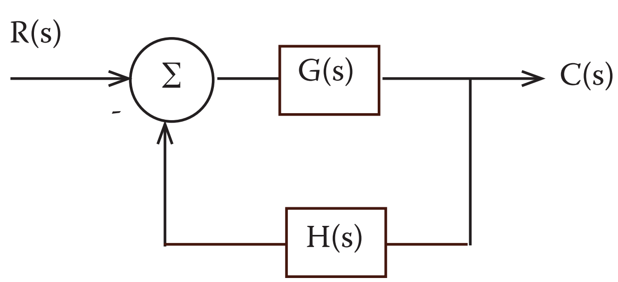
Simiarly, suppose we have a function of three variables, $f(x, y, z)$. We still write:

$$\Delta f = \frac{\partial f}{\partial x}\Delta x + \frac{\partial f}{\partial y}\Delta y + \frac{\partial f}{\partial z}\Delta z$$

We use this result over and over again when we're working with nonlinear systems.

Point of emphasis from lecture: "Trading Gain for Desensitivity"

Let's go over this slowly...



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So how sensitive is the overall transfer function to changes in G? Using our Taylor Series Approximation, we say

$$\begin{aligned}\Delta\left(\frac{C}{R}\right) &= \frac{d}{dG}\left(\frac{G}{1+GH}\right)\cdot \Delta G \\ &= \left[G\cdot\left(-\frac{H}{(1+GH)^2}\right) + \frac{1}{1+GH}\right]\cdot\Delta G \\ &= \left[-\frac{GH}{(1+GH)^2}) + \frac{1}{1+GH}\right]\cdot\Delta G \\ &= \frac{1}{(1+GH)^2}\,\Delta G\end{aligned}$$

More useful to us is knowing the fractional change in the transfer function given a fractional change in G. This allows us to say things like, “A 10% change in G leads to only a 0.1% change in the overall transfer characteristic.” So we divide both sides of our result by C/R:

$$\frac{\Delta(C/R)}{C/R} = \frac{\frac{1}{(1+GH)^2}\,\Delta G}{C/R}$$

But C/R = $\frac{G}{1+GH}$ ...

$$\frac{\Delta(C/R)}{C/R} = \frac{\frac{1}{(1+GH)^2}\,\Delta G}{\frac{1}{1+GH}}$$

$$\frac{\Delta(C/R)}{C/R} = \frac{1}{1+GH}\quad\frac{\Delta G}{G}$$

For large GH (GH>>1), we get

$$\frac{\Delta(C/R)}{C/R} \approx \frac{1}{GH}\quad\frac{\Delta G}{G}$$

What do we now mean by "trading gain for desensitivity?" For large values of GH, the closed-loop TF is approximately 1/H. So, in some sense, the only gain that we really "need" could be provided by an open-loop amplifier with gain 1/H. But the forward path gain that we have is G. The amount of gain, then, that we're "throwing away" is

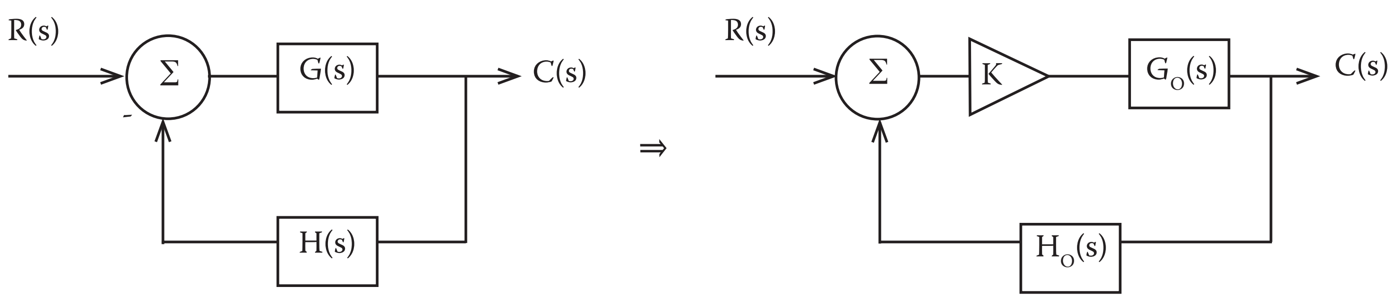
$$\frac{\text{Original forward path gain}}{\text{Gain that we need}} = \frac{G}{1/H} = GH$$

We can call GH the excess gain, or the gain that we've thrown away. But look: the gain that we've thrown away shows up again as a reduction of sensitivity to variations in G.

$$\frac{\Delta/(C/R)}{C/R} \approx \frac{1}{GH} \frac{\Delta G}{G}$$

Spend some time thinking about this.

There is often an implicit assumption in our analyses that our block diagrams can be drawn in the following way:



$L(s) = G(s)H(s)$ $\qquad$ $L_O(s) = \frac{L(s)}{K}$

$L_O(S)$ is the normalized loop transmission.

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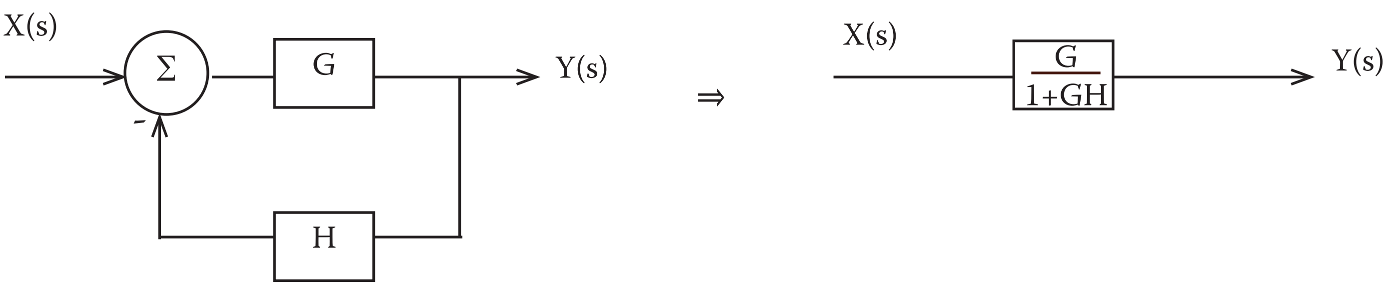
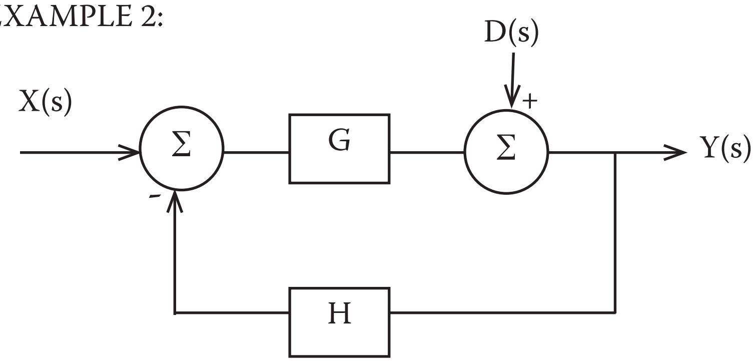
Now we move on to an important skill, which is manipulating block diagrams. One of the most useful tricks you will use is "collapsing" a feedback loop using Black's Formula:

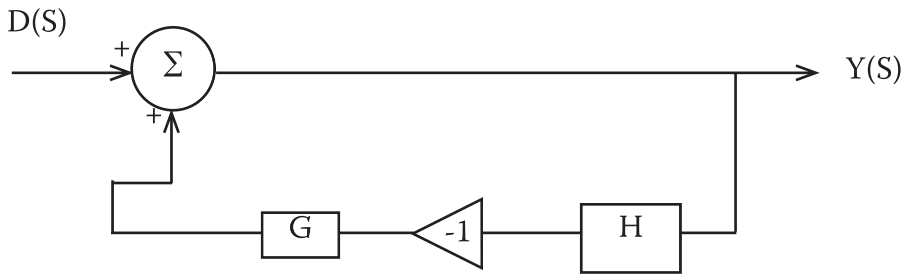
EXAMPLE 1:



EXAMPLE 2:



What is the transfer function Y(s)/D(s)? Redraw:



Easy now!

$$\frac{Y(S)}{D(S)} = \frac{1}{1+GH}$$

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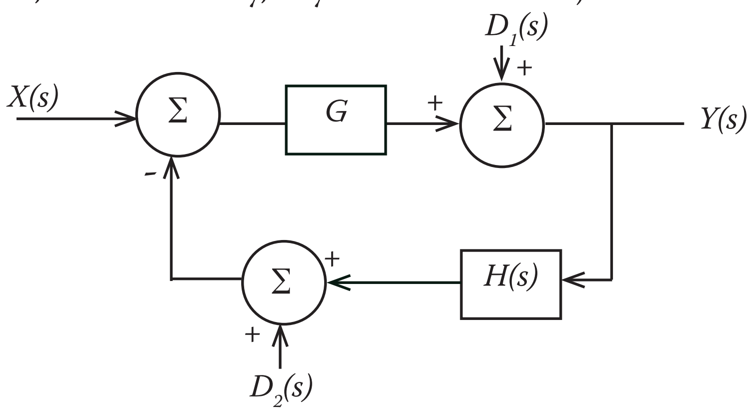
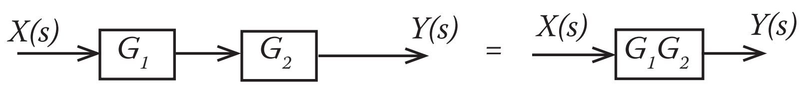
CLASS EXERCISE

In the following diagram, which distrubance bothers us the most? (HINT: Block diagram manipulation & Black's Formula is one, but not the only, way to reason this out.)



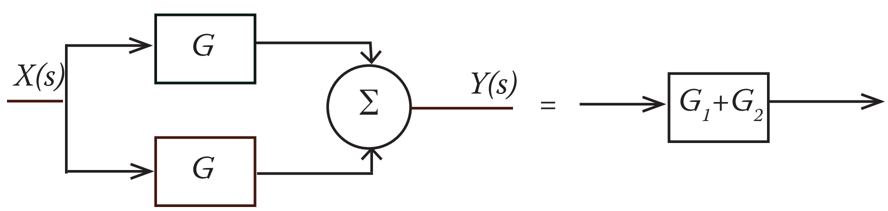
<u>Other useful results</u>: (graphical expressions of <u>math</u>)

1. Block in series:



$y(t) = x(t) * g_1(t) * g_2(t)$ $\xrightarrow[\text{TRANSFORM}]{\text{LAPLACE}}$ $Y(S) = G_1(s)G_2(s)X(s)$

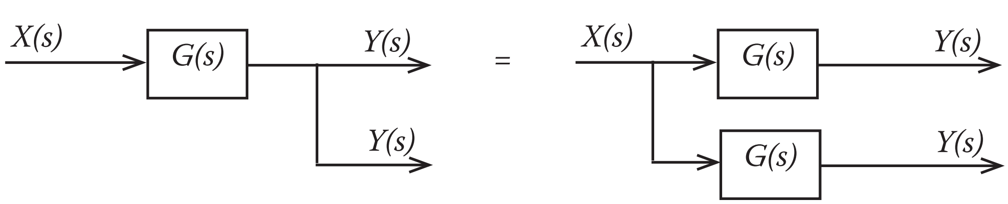
2. Blocks in parallel:



$Y(s) = G_1(s)X(s) + G_2(s)X(s)$

$= (G_1(s) + G_2(s))X(s)$

3. Blocks moved through a junction:



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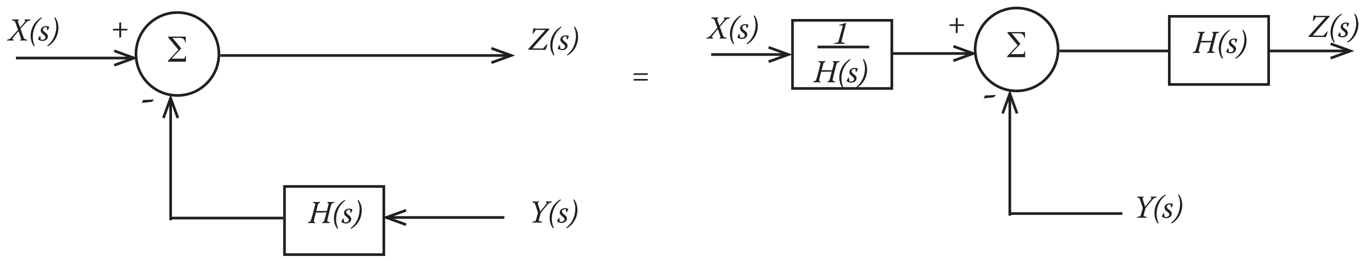
(4) Moving blocks through a summer:



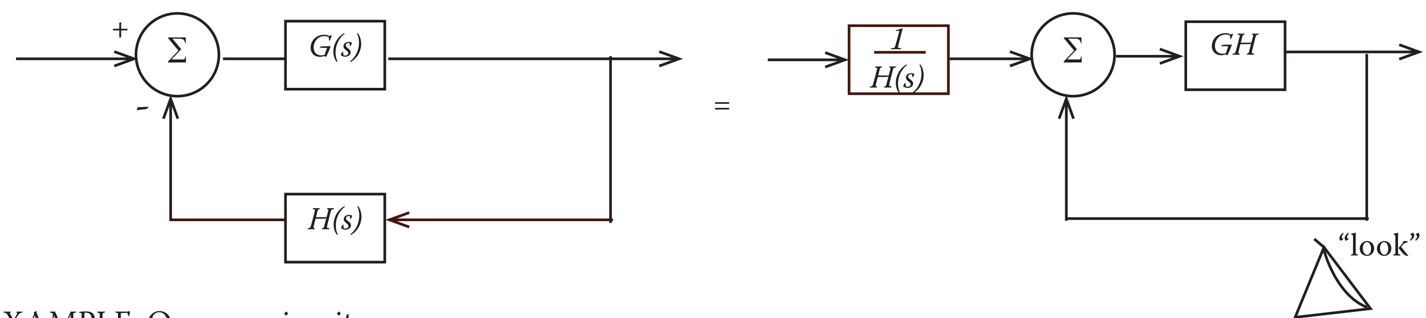
VERY USEFUL. Why does it work? Start with

$$(X - HY) = Z$$

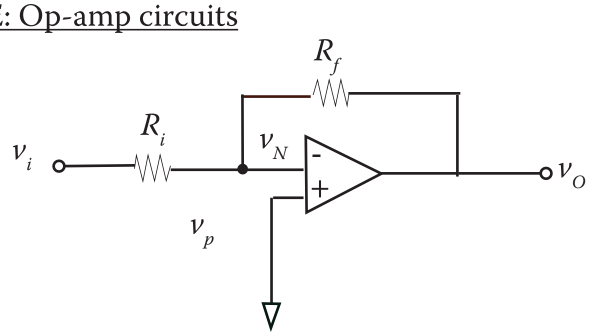
$$H \cdot 1/H\,(X - HY) = Z$$

$$H\left(\tfrac{1}{H}X - Y\right) = Z \qquad \text{DONE.}$$

Useful because it lets us draw unity feedback diagrams quickly:



EXAMPLE: Op-amp circuits



$$v_O = A(S) \cdot [\cancel{v_p} - v_N] = -A(S)v_N$$

must figure out v_N: ⟹

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Recitation 3: Block Diagrams (continued)

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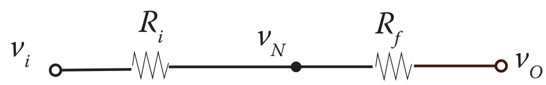
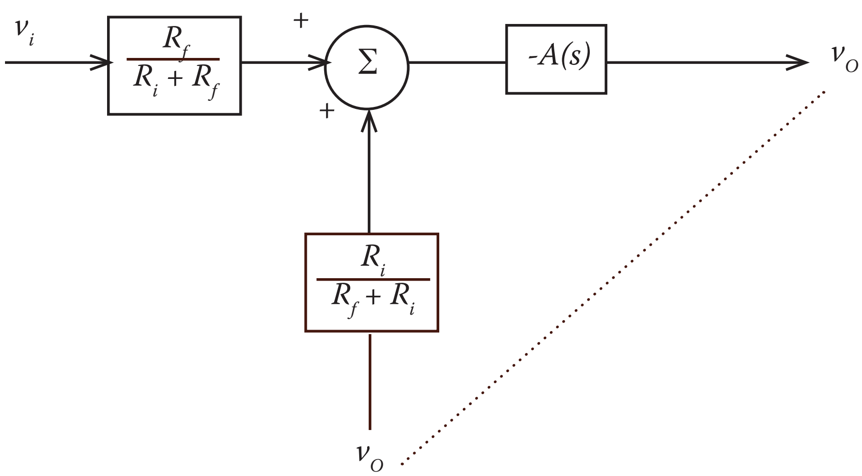
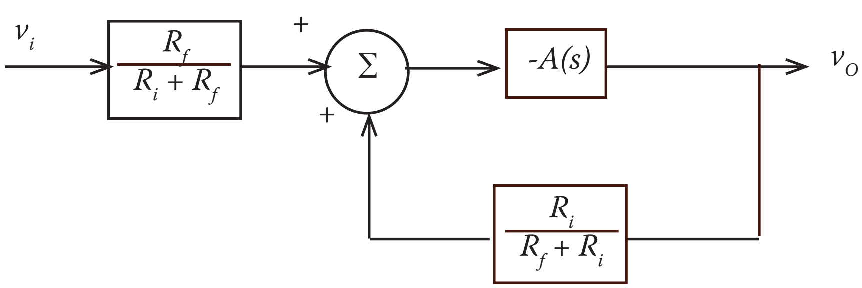


using superposition:

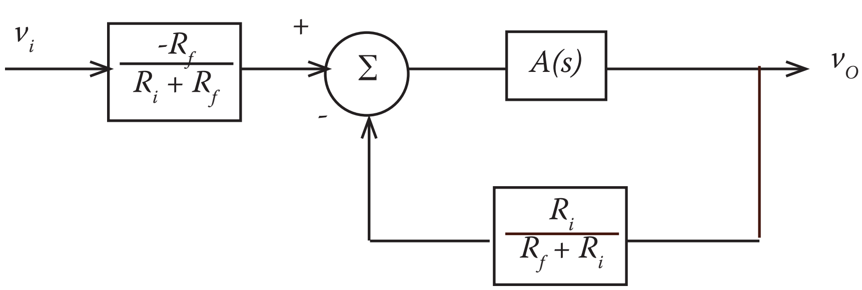
$$v_N = \frac{R_i}{R_i + R_f} v_O + \frac{R_f}{R_i + R_f} v_i$$

Block diagram; start with a summing junction =>





Now move minus sign through summing junction:



and draw as unity feedback: ⇒