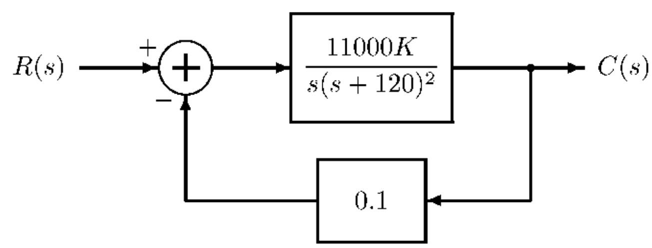
MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Electrical Engineering and Computer Science

6.302 Feedback Systems

Spring Term 2007 — Issued : February 21, 2007
Problem Set 3 — Due : Tuesday, February 27, 2007

Problem 1: An op amp is connected in an inverting configuration to provide an "ideal" gain of -10. The step response of the closed-loop system is approximately first order with a 10–90% rise time of 2.2 μs. Estimate the open-loop transfer function $A(s)$ of the op amp.

Problem 2: Consider a feedback system



For parts (a), (b), (c), and (d) assume $K = 110$.

(a) Calculate the closed-loop transfer function $C/R(s)$. Find the locations of the closed-loop poles. Is this system better approximated by a first-order system or a second-order system?

(b) Based on the pole locations, find the "approximation parameters" (that is, find τ if the system is best approximated by a first-order system, or find ω_n and ζ if the system is best approximated by a second-order system).

(c) Use Matlab to plot the step response. From a hardcopy of the plot, find P_o, t_p, and t_r. From these "measurements," calculate the approximation parameters.

(d) Use Matlab to plot the Bode plot. From a hardcopy of the plot, find M_p, ω_p, and ω_h. From these "measurements," calculate the approximation parameters.

(e) Repeat part (a) for $K = 2000$.

(f) Repeat part (b) for $K = 2000$.

(g) Repeat part (c) for $K = 2000$.

(h) Repeat part (d) for $K = 2000$.

Problem 3: A unity-feedback system has a loop transfer function given by

$$L(s) = \frac{K}{s(0.5s+1)}$$

(a) For a unit-step input, the closed-loop response must have a peak overshoot of 16%. Determine the required value of K.

(b) If $K = 2$, what is the approximate rise time of the output response to a step input?

(c) If $K = 2$ and the input is a unit ramp, what is the steady-state error?

Problem 4: *For the block diagrams given in Figures 1 and 2, compare the effect of the location of the* integrator on the stability and steady-state errors for step and ramp inputs of both input R and D. Express and compare all of the steady-state errors:

(a) How do these errors for R and D compare for the system shown in Figure 1?

(b) How do these errors compare for the system shown in Figure 2?

(c) Explain these differences in behavior.

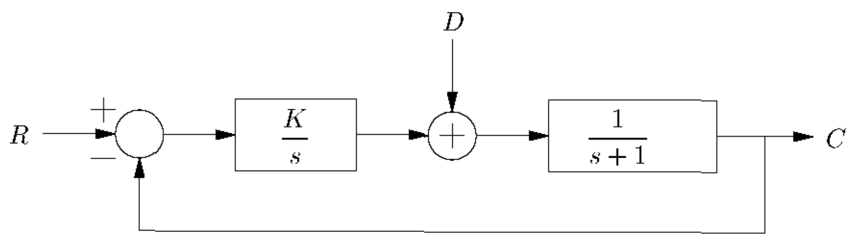


Figure 1: Steady-state error system 1

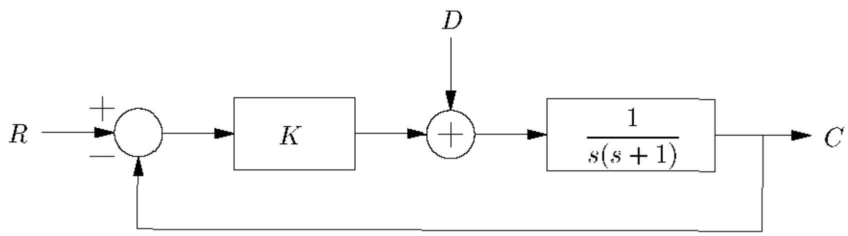


Figure 2: Steady-state error system 2

Problem 5: Consider the following loop transfer functions:

$$L_1(s) = \frac{10^6}{s} \qquad L_2(s) = \frac{10^6}{s+1} \qquad L_3(s) = \frac{10^{10}(10^{-4}s+1)}{s^2} \qquad L_4(s) = \frac{10^6(10^{-4}s+1)}{(10^{-2}s+1)^2}$$

For each loop transfer function:

(a) Plot an asymptotic Bode Plot.

(b) Find the error transfer function, assuming that the above loop transfer functions describe opamp circuits with unity feedback.

(c) Find the steady state error to a 1 V step input.

(d) Find the steady state error to a 1 V/s ramp input.

Problem 6: A linear system may be stable for certain inputs and unstable for other inputs. True or false? Justify your answer.

Problem 7: This should be completed using Octave, MATLAB or similar software. You may find it helpful to save your work as it may be useful in the future. Please hand in clearly labelled printouts.

The purpose of this problem is to investigate the relationship between time and frequency responses of linear systems. Produce a table with the transfer function and expressions for the rise time, the bandwidth, magnitude peaking, peaking frequency, peak overshoot, time to peak, and the 2% settling time for the following linear systems.

First order system.

$$H_1(s) = \frac{1}{\tau s + 1}$$

Second order system.

$$H_2(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n + \omega_n^2}$$

Now produce the required plots for the Second order system only, with the following set of values:

- ω_n held constant at 5 rps and five values of ζ from 0.1 to 1 including 0.707.
- $\zeta = 0.5$ and five values of ω_n from 2 to 50 (excluding $\omega_n = 5$).
- $\zeta\omega_n = 2.5$ for five different values of ζ or ω_n (that is, if $\zeta = 0.2$, $\omega_n = 2.5/0.2 = 12.5$)

You should now have 15 $\zeta - \omega_n$ pairs. With these values produce the following plots (first item listed should be on the x-axis). Each plot should have 15 data points.

1. ζ vs. Magnitude Peaking.
2. Magnitude Peaking vs. Peak Overshoot.
3. Rise Time vs. Bandwidth.

Problem 8: Match each step response (1–8) to the appropriate Bode magnitude plot (A–H).