Lecture 19

Broadcast routing

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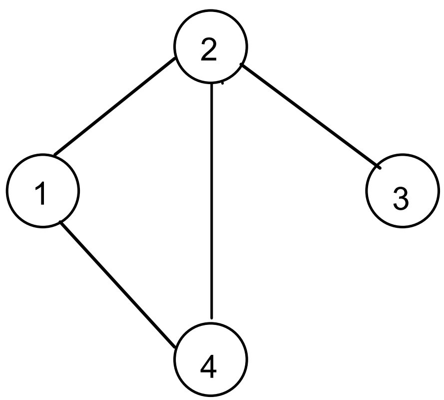
Broadcast Routing

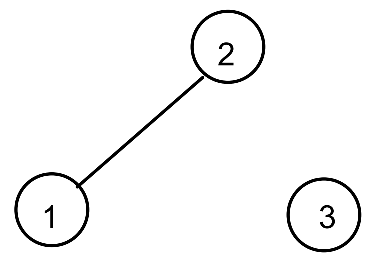
- **Route a packet from a source to all nodes in the network**
- **Possible solutions:**
  - **Flooding: Each node sends packet on all outgoing links**
    Discard packets received a second time
  - **Spanning Tree Routing: Send packet along a tree that includes all of the nodes in the network**

Graphs

- **A graph G = (N,A) is a finite nonempty set of nodes and a set of node pairs A called arcs (or links or edges)**



N = {1,2,3,4}
A = {(1,2),(2,3),(1,4),(2,4)}

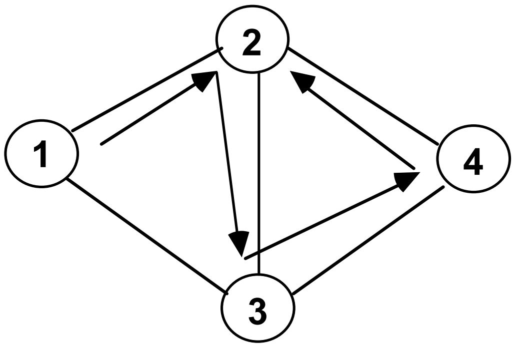


N = {1,2,3}
A = {(1,2)}

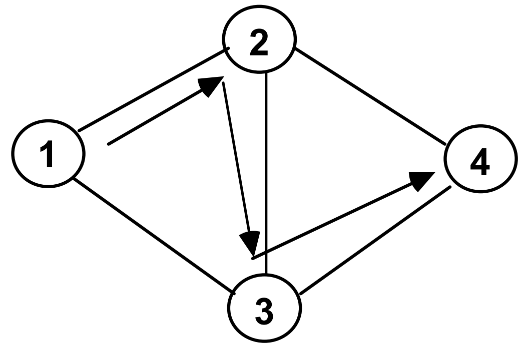
Walks and paths

- A walk is a sequence of nodes (n1, n2, ...,nk) in which each adjacent node pair is an arc.

- A path is a walk with no repeated nodes.



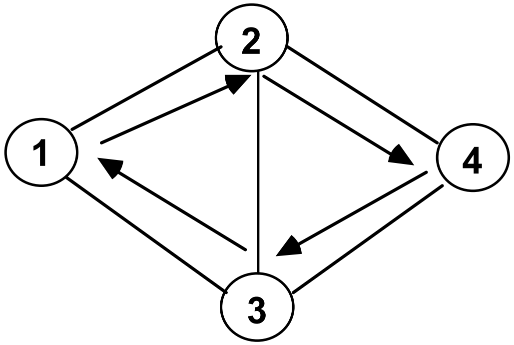
Walk (1,2,3,4,2)



Path (1,2,3,4)

Cycles

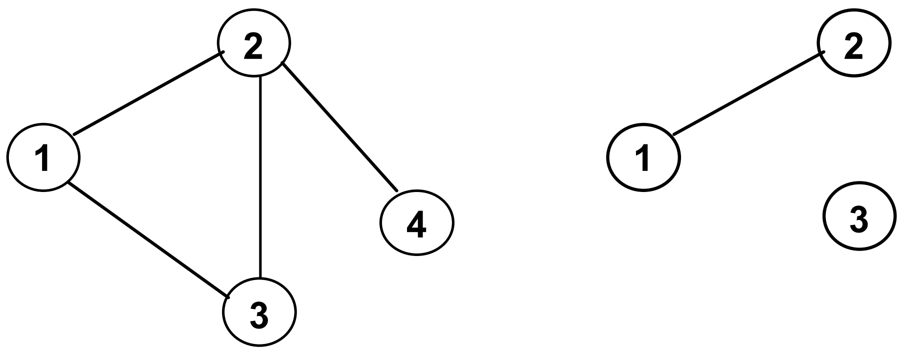
- **A cycle is a walk (n1, n2,...,nk) with n1 = nk, k>3, and with no repeated nodes except n1 = nk**



Cycle (1,2,4,3,1)

Connected graph

- A graph is connected if a path exists between each pair of nodes.

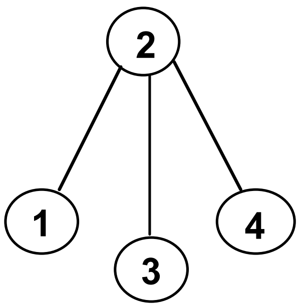


Connected Unconnected

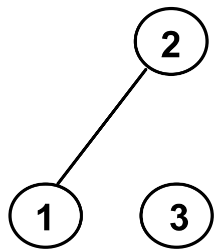
- An unconnected graph can be separated into two or more connected components.

Acyclic graphs and trees

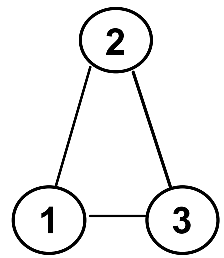
- An acyclic graph is a graph with no cycles.
- A tree is an acyclic connected graph.



Acyclic,
connected



unconnected
not tree



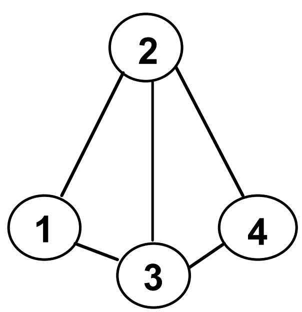
Cyclic,
not tree

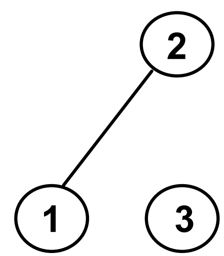
- The number of arcs in a tree is always one less than the number of nodes
  - Proof: start with arbitrary node and each time you add an arc you add a node => N nodes and N-1 links. If you add an arc without adding a node, the arc must go to a node already in the tree and hence form a cycle

Subgraphs

- G' = (N',A') is a subgraph of G = (N,A) if
  - 1) G' is a graph
  - 2) N' is a subset of N
  - 3) A' is a subset of A

- One obtains a subgraph by deleting nodes and arcs from a graph
  - Note: arcs adjacent to a deleted node must also be deleted

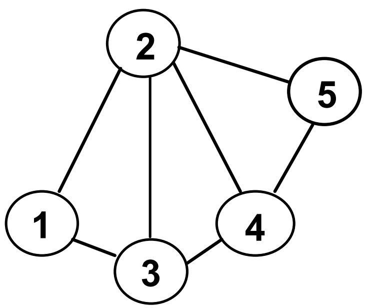




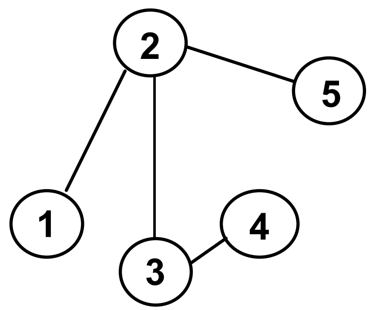
– Graph G Subgraph G' of G

Spanning trees

- T = (N',A') is a spanning tree of G = (N,A) if
  - T is a subgraph of G with N' = N and T is a tree



Graph G



Spanning tree of G

Spanning trees

- Spanning trees are useful for disseminating and collecting control information in networks; they are sometimes useful for routing

- To disseminate data from Node n:
  - Node n broadcasts data on all adjacent tree arcs
  - Other nodes relay data on other adjacent tree arcs

- To collect data at node n:
  - All leaves of tree (other than n) send data
  - Other nodes (other than n) wait to receive data on all but one adjacent arc, and then send received plus local data on remaining arc

General construction of a spanning tree

- **Algorithm to construct a spanning tree for a connected graph G = (N,A):**

  1) Select any node n in N; N' = {n}; A' = { }

  2) If N' = N, then stop (T=(N',A') is a spanning tree)

  3) Choose $(i,j) \in A$, $i \in N'$, $j \notin N'$

  $N' := N' \cup \{j\}$; $A' := A' \cup \{(i,j)\}$; go to step 2

- **Connectedness of G assures that an arc can be chosen in step 3 as long as $N' \neq N$**

- **Is spanning tree unique?**

Spanning tree algorithm

- The algorithm never forms a cycle, since each new arc goes to a new node.
- T = (N',A') is a tree at each step of the algorithm since T is always connected, and each time we add an arc we also add a node
- Theorem: If G is a connected graph of n nodes, then

  1) G contains at least n-1 arcs

  2) G contains a spanning tree

  3) if G contains exactly n-1 arcs, G is a spanning tree

Distributed algorithms to find spanning trees

1) A fixed node sends a "start" message on each adjacent arc of the graph

2) Each other node marks the first arc on which a start message was received as a spanning tree arc and then sends a "start" message on each other arc

– This is a distributed implementation of the general spanning tree algorithm

– It has several problems shared by many such algorithms:

a) who chooses the starting node?
b) When does the algorithm terminate?
c) The resulting tree is somewhat random

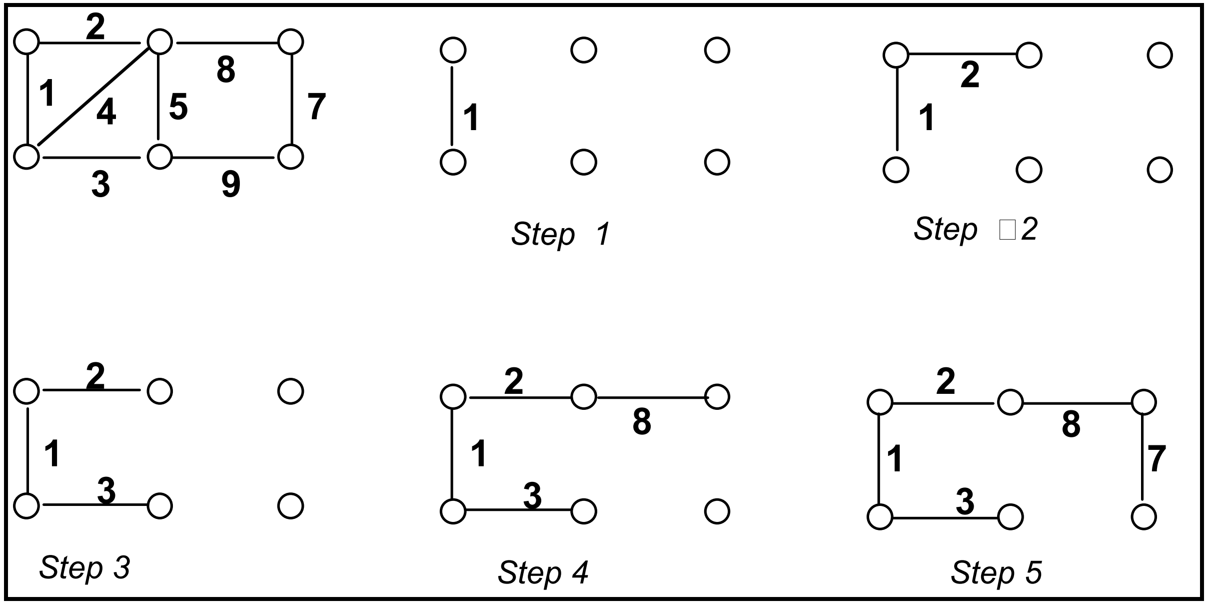
Min weight spanning tree

- **Given a graph with weights assigned to each arc, find a spanning tree of minimum total weight (MST)**
- **Define a "fragment" to be a subtree of a MST**
- **Theorem:**
  - **Given a fragment F of an MST, Let a(i,j) be a minimum weight outgoing arc from F, where j is not in F.**
  - **Then, F extended by arc a(i,j) & node j is a fragment.**
- **Proof:**
  - **Let M be the MST that does not include a(i,j).**
  - **Since a(i,j) is not part of M, then adding a(i,j) to M must cause a cycle. There must be some link in the cycle b ≠ a which is outgoing from F.**
  - **Deleting b and adding a creates a new spanning tree. Since weight of b cannot be less then weight of a , M' must be a MST.**
    - **If weight of a = weight of b, then both are MST's otherwise M could not have been an MST**

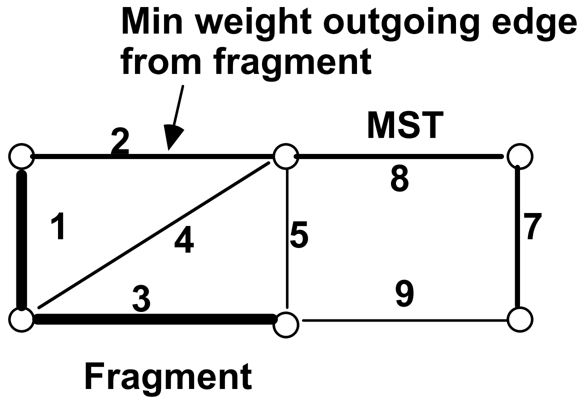
MST algorithms

- Generic MST algorithm steps:
  - Given a collection of subtrees of an MST (called fragments) add a minimum weight outgoing edge to some fragment
- Prim-Dijkstra: Start with an arbitrary single node as a fragment
  - Add minimum weight outgoing edge
- Kruskal: Start with each node as a fragment;
  - Add the minimum weight outgoing edge, minimized over all fragments

Prim-Dijkstra Algorithm



Kruskal Algorithm



- **Suppose the arcs of weight 1 and 3 are a fragment**
  - **Consider any spanning tree using those arcs and the arc of weight 4, say, which is an outgoing arc from the fragment.**
  - **Suppose that spanning tree does not use the arc of weight 2.**
  - **Removing the arc of weight 4 and adding the arc of weight 2 yields another tree of smaller weight.**
  - **Thus an outgoing arc of min weight from fragment must be in MST.**