Appendix A

Table of standard random variables

The following tables summarize the properties of some common random variables. If a density or PMF is specified only in a given region, it is assumed to be zero elsewhere. The parameters λ, σ, and a are assumed to be positive, $p \in (0,1)$, and n is a positive integer.

Name	**Density** $\mathsf{f}_X(x)$	**Mean**	**Variance**	**MGF**
Exponential:	$\lambda \exp(-\lambda x);\; x{\ge}0$	$\frac{1}{\lambda}$	$\frac{1}{\lambda^2}$	$\frac{\lambda}{\lambda-r};\;$ for $r < \lambda$
Erlang:	$\frac{\lambda^n x^{n-1} \exp(-\lambda x)}{(n-1)!};\quad x{\ge}0$	$\frac{n}{\lambda}$	$\frac{n}{\lambda^2}$	$\left(\frac{\lambda}{\lambda-r}\right)^n;\;$ for $r < \lambda$
Gaussian:	$\frac{1}{\sigma\sqrt{2\pi}} \exp\left(\frac{-(x-a)^2}{2\sigma^2}\right)$	a	σ^2	$\exp(ra + r^2\sigma^2/2)$
Uniform:	$\frac{1}{a};\; 0{\le}x{\le}a$	$\frac{a}{2}$	$\frac{a^2}{12}$	$\frac{\exp(ra)-1}{ra}$

Name	**PMF** $\mathsf{p}_M(m)$	**Mean**	**Variance**	**MGF**
Binary:	$\mathsf{p}_M(1) = p;\; \mathsf{p}_M(0) = 1-p$	p	$p(1-p)$	$1 - p + pe^r$
Binomial:	$\binom{n}{m} p^m (1-p)^{n-m};\; 0{\le}m{\le}n$	np	$np(1-p)$	$[1 - p + pe^r]^n$
Geometric:	$p(1-p)^{m-1};\; m{\ge}1$	$\frac{1}{p}$	$\frac{1-p}{p^2}$	$\frac{pe^r}{1-(1-p)e^r};\;$ for $r < \ln \frac{1}{1-p}$
Poisson:	$\frac{\lambda^n \exp(-\lambda)}{n!};\; n{\ge}0$	λ	λ	$\exp[\lambda(e^r - 1)]$

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