6.262 Discrete Stochastic Processes
MIT, Spring 2011

Wednesday, March 9, 2011

Due: Wednesday, March 16

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**Problem Set 6**

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Reading Assignment: Read Chapter 4, Sections 4.1 to 4.7.1

1) a) Let $\{Y_n;\, n \geq 1\}$ be a sequence of rv's and assume that $\lim_{n\to\infty} \mathsf{E}|Y_n| = 0$. Show that $\{Y_n;\, n \geq 1\}$ converges to 0 in probability. Hint 1: Look for the easy way. Hint 2: The easy way uses the Markov inequality.

2) Exercise 4.2 in text.

3) Exercise 4.4 in text.

4) Exercise 4.5 in text.

5) Exercise 4.8 in text.

6) a) Suppose that an ergodic Markov chain with $\mathsf{M}$ states is started at time 0 in steady state, with probabilities $\pi_1, \dots, \pi_{\mathsf{M}}$. Consider a reward process in which unit reward occurs on each entry (at positive integer times) to state 1 and 0 reward occurs otherwise. Find $\mathsf{E}N_s(t)$ for positive integers $t$ where $N_s(t)$ is the accumulated reward up to and including time $t$. Hint: Reason directly from first principles — no need for Section 3.5.

Let $N_1(t)$ be the renewal process where a renewal occurs at each entry to state 1 and where $X_0 = 1$. Argue why $\lim_{t\to\infty} N_1(t)/t = \lim_{t\to\infty} N_s(t)/t = \lim_{t\to\infty} \mathsf{E}N_s(t)/t$ (no need for rigor here)

b) Find the expected inter-renewal time between successive entries to state 1. Hint: Use the strong law for renewals.

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