LECTURE SLIDES ON NONLINEAR PROGRAMMING

BASED ON LECTURES GIVEN AT THE

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

CAMBRIDGE, MASS

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These lecture slides are based on the book: “Nonlinear Programming,” Athena Scientific, by Dimitri P. Bertsekas; see

http://www.athenasc.com/nonlinbook.html

6.252 NONLINEAR PROGRAMMING

LECTURE 1: INTRODUCTION

LECTURE OUTLINE

- Nonlinear Programming
- Application Contexts
- Characterization Issue
- Computation Issue
- Duality
- Organization

NONLINEAR PROGRAMMING

$$\min_{x \in X} f(x),$$

where

- $f : \Re^n \mapsto \Re$ is a continuous (and usually differentiable) function of n variables
- $X = \Re^n$ or X is a subset of $\Re^n$ with a "continuous" character.

- If $X = \Re^n$, the problem is called unconstrained
- If f is linear and X is polyhedral, the problem is a linear programming problem. Otherwise it is a nonlinear programming problem
- Linear and nonlinear programming have traditionally been treated separately. Their methodologies have gradually come closer.

TWO MAIN ISSUES•

- Characterization of minima
  - Necessary conditions
  - Sufficient conditions
  - Lagrange multiplier theory
  - Sensitivity
  - Duality
- Computation by iterative algorithms
  - Iterative descent
  - Approximation methods
  - Dual and primal-dual methods

APPLICATIONS OF NONLINEAR PROGRAMMING•

- Data networks – Routing
- Production planning
- Resource allocation
- Computer-aided design
- Solution of equilibrium models
- Data analysis and least squares formulations
- Modeling human or organizational behavior

CHARACTERIZATION PROBLEM

- Unconstrained problems
  - Zero 1st order variation along all directions
- Constrained problems
  - Nonnegative 1st order variation along all feasible directions
- Equality constraints
  - Zero 1st order variation along all directions on the constraint surface
  - Lagrange multiplier theory
- Sensitivity

COMPUTATION PROBLEM

- Iterative descent
- Approximation
- Role of convergence analysis
- Role of rate of convergence analysis
- Using an existing package to solve a nonlinear programming problem

POST-OPTIMAL ANALYSIS

- Sensitivity
- Role of Lagrange multipliers as prices

DUALITY

- Min-common point problem / max-intercept problem duality

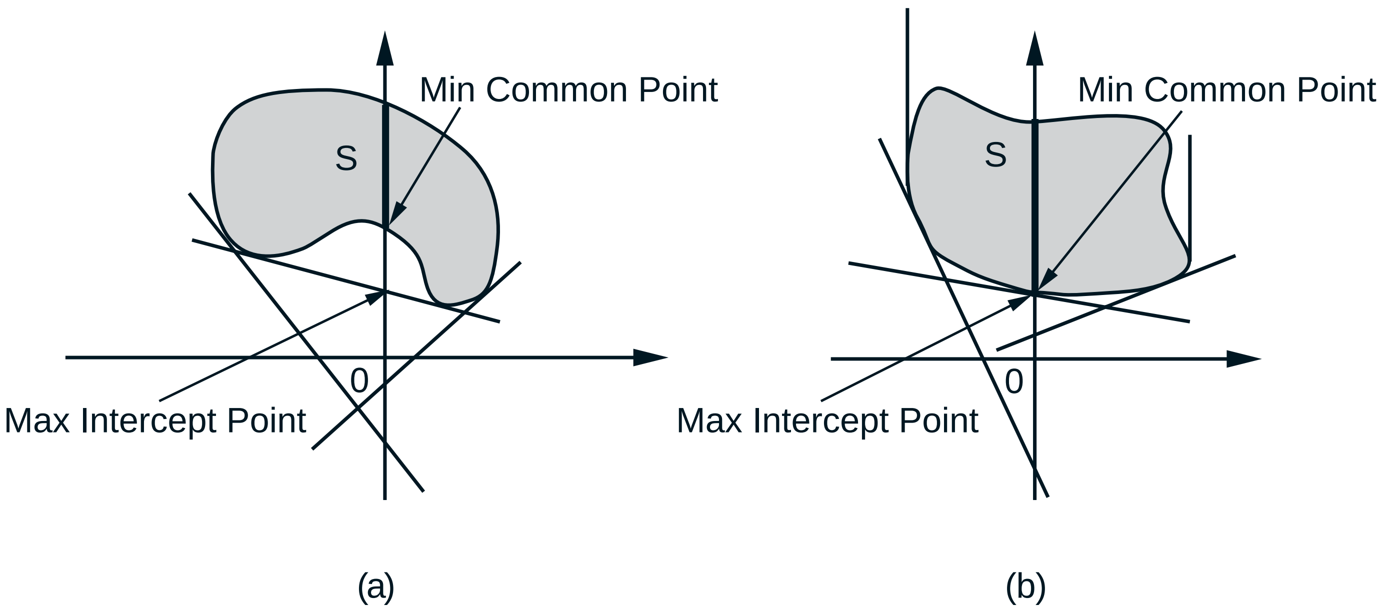

Illustration of the optimal values of the min common point and max intercept point problems. In (a), the two optimal values are not equal. In (b), the set S, when "extended upwards" along the nth axis, yields the set

$$\bar{S} = \{\bar{x} \mid \text{for some } x \in S,\, \bar{x}_n \geq x_n,\, \bar{x}_i = x_i, i = 1, \ldots, n-1\}$$

which is convex. As a result, the two optimal values are equal. This fact, when suitably formalized, is the basis for some of the most important duality results.