15.081 Fall 2009 Recitation for Lectures {9,10,11} Duality theory

Duality is the most important characteristic of LP. LP is useful and easy only because of the existence of strong duality. As one can observe, whenever we find a system where we find strong duality we can apply many of the algorithms that we develop for LPs for those problems. In particular, I recommend the following book for people interested in Duality theory. Duality in Optimization

1 Concepts

Primal and Dual

- 1. Lagrangean Duality: The duality theory developed arises from consideing the lagrangean relaxation of the LP.
- 2. Weak Duality : Given any dual feasible p and primal feasible x suc that $p'b \leq c'x$.
- 3. Strong Duality and Complementary Slackness The properties of optimal solutions of primal and dual.

Geometry

- 1. Farkas Lemma, which turns out to be equivalent to Strong Duality.
- 2. Cones and Extreme Rays Extreme rays of a polyhedron correspond to the extreme rays of the recession cone associated with the polyhedron.
- 3. Resolution theorem: Another representation of a polyhedron.

Duality and Degeneracy

An important relation that can be useful is the following:

Uniqueness and Nondegeneracy of primal ←⇒ Uniqueness and Nondegeneracy of the dual.

This would thus imply that :

• Non-uniqueness or degenracy of primal \iff Non-uniqueness or degenracy of the dual.

Proof Techniques

The proof techniques that duality or Farkas' Lemma provides us with are very useful. In particuar, duality allows us to convert an "existential statement" into a "for all statement". Whenever we have an "existential statement" in the hypothesis, duality would give us "for all statements" as conclusions and vice-versa. Most of the exercises are (slighlty non-trivial) applications of this proof technique.

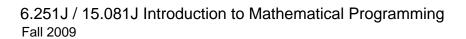
In particular, to use duality we have identify

- Appropriate Primal Polyhedron This usually is very apparent from the statement of the problem that we are considering.
- Appropriate cost function this requires "clever" choice and usually choices of $c=0,\pm e,\pm e_i$ work.

Examples:

- 1. Strict complementary slackness (4.20) The polyhderon and the cost vector suggested in the hint.
- 2. Clarke's theorem Polyhedron is obvious. cost vector Use -e.
- 3. 4.26: USe e





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