# 15.081 Fall 2009 Recitation for Lectures {9,10,11} Duality theory

Duality is the most important characteristic of LP. LP is useful and easy only because of the existence of strong duality. As one can observe, whenever we find a system where we find strong duality we can apply many of the algorithms that we develop for LPs for those problems. In particular, I recommend the following book for people interested in Duality theory. Duality in Optimization

# 1 Concepts

#### Primal and Dual

- 1. Lagrangean Duality: The duality theory developed arises from consideing the lagrangean relaxation of the LP.
- 2. Weak Duality : Given any dual feasible p and primal feasible x suc that  $p'b \leq c'x$  .
- 3. Strong Duality and Complementary Slackness The properties of optimal solutions of primal and dual.

#### Geometry

- 1. Farkas Lemma, which turns out to be equivalent to Strong Duality.
- 2. Cones and Extreme Rays Extreme rays of a polyhedron correspond to the extreme rays of the recession cone associated with the polyhedron.
- 3. Resolution theorem: Another representation of a polyhedron.

## **Duality and Degeneracy**

An important relation that can be useful is the following:

Uniqueness and Nondegeneracy of primal ←⇒ Uniqueness and Nondegeneracy of the dual.

This would thus imply that :

• Non-uniqueness or degenracy of primal  $\iff$  Non-uniqueness or degenracy of the dual.

# **Proof Techniques**

The proof techniques that duality or Farkas' Lemma provides us with are very useful. In particuar, duality allows us to convert an "existential statement" into a "for all statement". Whenever we have an "existential statement" in the hypothesis, duality would give us "for all statements" as conclusions and vice-versa. Most of the exercises are (slighlty non-trivial) applications of this proof technique.

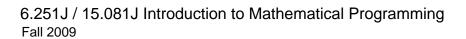
In particular, to use duality we have identify

- Appropriate Primal Polyhedron This usually is very apparent from the statement of the problem that we are considering.
- Appropriate cost function this requires "clever" choice and usually choices of  $c=0,\pm e,\pm e_i$  work.

## Examples:

- 1. Strict complementary slackness (4.20) The polyhderon and the cost vector suggested in the hint.
- 2. Clarke's theorem Polyhedron is obvious. cost vector Use -e.
- 3. 4.26: USe e





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