Massachusetts Institute of Technology

Department of Electrical Engineering and Computer Science

6.245: MULTIVARIABLE CONTROL SYSTEMS

by A. Megretski

Problem Set 7 (due April 7, 2004) [1]

Problem 7.1

Consider the system described by Figure 7.1, where $P(s) = 1/(s+a)$, and $a \in \mathbf{R}$, $\epsilon > 0$ are parameters.

(a) Find analytically (as a function of $a \in \mathbf{R}$ and $\epsilon > 0$) the maximal lower bound $\gamma = \gamma(a, \epsilon)$ of the H-Infinity norm of the transfer matrix from $w = [w_1; w_2]$ to $z = [z_1; z_2]$, achievable while using a stabilizing LTI controller $C = C(s)$.

(b) For $a = 1, \epsilon = 1$ and for all $\delta > 0$ find analytically (i.e. as a function of $\delta > 0$) a controller $C = C_\delta(s)$ which makes the H-Infinity norm of the closed loop transfer matrix from w to z less than $\gamma(1, 1) + \delta$.

(c) Check your solution sing MATLAB calculations.

Problem 7.2

Let $\mathcal{H}_G$ be the Hankel operator associated with the transfer function

$$G(s) = \frac{1}{s+a} + \frac{1}{s+2a},$$

where $a > 0$ is a positive parameter.

[1] Version of March 31, 2004

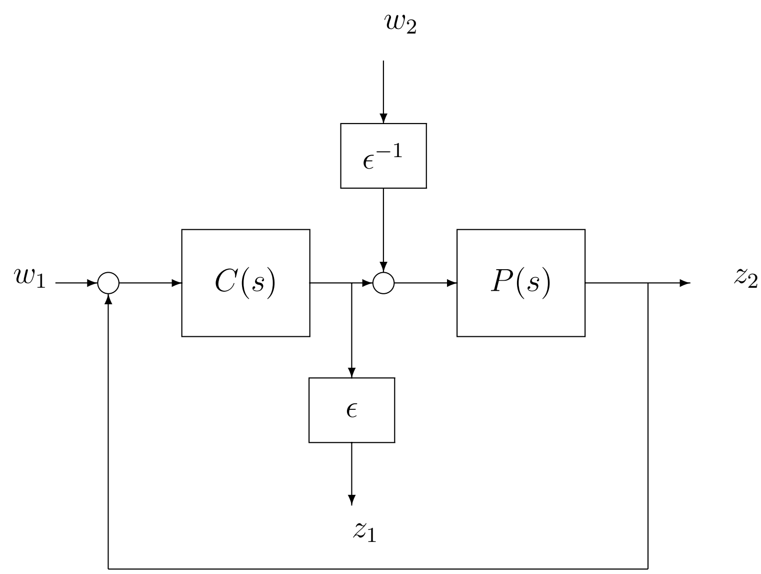

Figure 7.1: Diagram for Problem 7.1

(a) Give an analytical expression (as functions of $a > 0$) for all non-zero singular values of $\mathcal{H}_G$, as well as for the corresponding singular vectors.

(b) Give an analytical expression for a Hankel optimal first order reduced model $\hat{G}$ of G.

(c) Check your solution using MATLAB calculations.

Problem 7.3

Infinite order transfer function G is defined by

$$G(s) = \frac{1}{s-1} \int_1^2 \frac{da}{s+a}.$$

Find a transfer function $\hat{G}$ of a order m (try to make m as small as possible) such that $G - \hat{G}$ is stable, and $\|G - \hat{G}\|_\infty < 0.02$. You are expected to use Hankel optimal model reduction (function `hankmr.m` of MATLAB), combined with approximation of the integral by a finite sum.