**Massachusetts Institute of Technology**

**Department of Electrical Engineering and Computer Science**

**6.245: MULTIVARIABLE CONTROL SYSTEMS**

**by A. Megretski**

# Problem Set 4 (due March 3, 2004) [1]

## Problem 4.1

For the SISO feedback design from Figure 4.1, where it is known that $P(2) = 0$ and $1 \pm 2j$ are poles of $P$, find a lower bound (as good as you can) on the H-Infinity norm of the closed-loop complementary sensitivity transfer function $T = T(s)$ (from $r$ to $v$), assuming that $C = C(s)$ is a stabilizing controller, $|T(j\omega) - 1| < 0.2$ for $|\omega| < 10$, and $|T(j\omega)| < 0.1$ for $|\omega| > 20$.

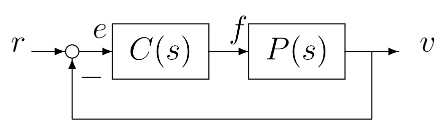


Figure 4.1: A SISO Feedback Setup

## Problem 4.2

(a) Apply the formulae for H2 optimization to a standard setup with a Hurwitz matrix $A$ and with $B_2 = 0$ to express the H2 norm of CT LTI MIMO state space model

$$y = Cx, \ \dot{x} = Ax + Bf$$

[1] Version of February 25, 2004

in terms of matrices $C$ and $P$, where $P = P'$ is the solution of the Lyapunov equation

$$AP + PA' = -BB'.$$

(b) Use the result from (a) to obtain an explicit (with respect to $a_0, a_1, a_2$) formula for the H2 norm of system with transfer function

$$G(s) = \frac{1}{s^3 + a_2 s^2 + a_1 s + a_0},$$

where $a_0, a_1, a_2$ are positive real numbers such that $a_1 a_2 > a_0$.

(c) Use MATLAB to check numerically correctness of your analytical solution.

**Problem 4.3**

An undamped linear oscillator with 3 degrees of freedom is described as a SISO LTI system with control input $v$, "position" output $q$, and transfer function

$$P_0(s) = \frac{1}{(1+s^2)(4+s^2)(16+s^2)}.$$

The position output $q$ is measured with delay and noise, so that the sensor output $g$ is defined as $g = 0.1 f_1 + \hat{q}$, where $\hat{q}$ is output of an LTI system with input $q$ and transfer function

$$P_1(s) = \frac{1-s}{1+s}.$$

The task is to design an LTI controller which takes $g$ and a scalar reference signal $r$ as inputs, produces $v$ as its output, and satisfies the following specifications, assuming $r$ is the output of an LTI system with input $f_2$ and transfer function

$$P_2(s) = \frac{\sqrt{2a}}{s+a},$$

where $a > 0$ is a parameter, and $f = [f_1; f_2]$ is a normalized white noise:

(a) the closed loop system is stable;

(b) the closed loop dc gain from r to q equals 1;

(c) the mean square value of control signal $v$ is within 100;

(d) the mean square value of tracking error $e = q - r$ is as small as possible (within 20 percent of its minimum subject to constraints (a)-(c)).

Use H2 optimization to solve the problem for $a = 0.2$ and $a = 0.01$.