Massachusetts Institute of Technology

Department of Electrical Engineering and Computer Science

6.245: MULTIVARIABLE CONTROL SYSTEMS

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Problem Set 2 (due February 18) [1]

Problem 2.1

For each of the statements below decide whether it is true or false. For a true statement, sketch a proof. For a false statement, give a counterexample.

(a) H2 norm of a stable finite order CT LTI state space model is never larger than 100 times its H-Infinity norm.

(b) H-Infinity norm of a stable finite order CT LTI state space model is never larger than 100 times its H2 norm.

(c) H2 norm of a stable finite order DT LTI state space model is never larger than 100 times its H-Infinity norm.

(d) H-Infinity norm of a stable finite order DT LTI state space model is never larger than 100 times its H2 norm.

Problem 2.2

For continuous time (non-LTI) systems S_a with scalar input $f = f(t)$ and scalar output $g = g(t)$, described below, find their L2 gains, as functions of parameter $a > 0$. Support your answer with arguments (typically, to show that L2 gain of a system equals, say, 0.5, one has to find input/output pairs of infinite energy (i.e. not converging to zero) for which

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the asymptotic (as time converges to infinity) output-to-input energy ratio is arbitrarily close to $0.25 = 0.5^2$, and, in addition, to show that the asymptotic energy ratio cannot be larger than 0.5.

(a) $g(t) = a\sin(f(t))$;

(b) $g(t) = f(at)\sin(t)$;

(c) $g(t) = af(t) - |f(t-1)|$.

Problem 2.3

Continuous time signal $q = q(t)$ is the output of a pure double integrator system with input $f_1 = f_1(t)$, and $g(t) = q(t) + bf_2(t)$, where $b > 0$ is a known constant (do the calculations for $b = 0.1$ and $b = 10$). Find an LTI filter $F = F(s)$ which takes $g = g(t)$ as an input and outputs an estimate $\hat{q} = \hat{q}(t)$ of $q = q(t)$, which is "good" in one of the following interpretations.

(a) Assuming that $f = [f_1; f_2]$ is white noise, minimize the asymptotic value of the variance of the estimation error $e = q - \hat{q}$.

(b) Minimize the L2 gain from $f = [f_1; f_2]$ to the estimation error $e = q - \hat{q}$ with accuracy 10 percent.

While there are several MATLAB programs available for solving this problem (and it can also be solved analytically), you are asked to find ways to apply `h2syn.m` and `hinfsyn.m` in this setting. To do this, you will have to resolve the "stabilizability" problem: the standard feedback optimization setup requires stabilizability, which does not take place here (as in many other state estimation problems). One way around this is to introduce *some* (non-optimal) state observer into the picture, and re-write system equations in terms of the state estimation error variables.