Massachusetts Institute of Technology

Department of Electrical Engineering and Computer Science

6.245: MULTIVARIABLE CONTROL SYSTEMS

by A. Megretski

Problem Set 1 [1]

Problem 1.1

Consider the task of finding a controller $F = F(s)$ (with two inputs r, q and one output v) which stabilizes the system on Figure 1.1 with

$$H(s) = \frac{10}{s+10}, \quad P_0(s) = \frac{s}{s^2+1},$$

and minimizes the H2 norm of the closed loop transfer function from f to e (essentially, this means minimizing the tracking error at low frequencies).

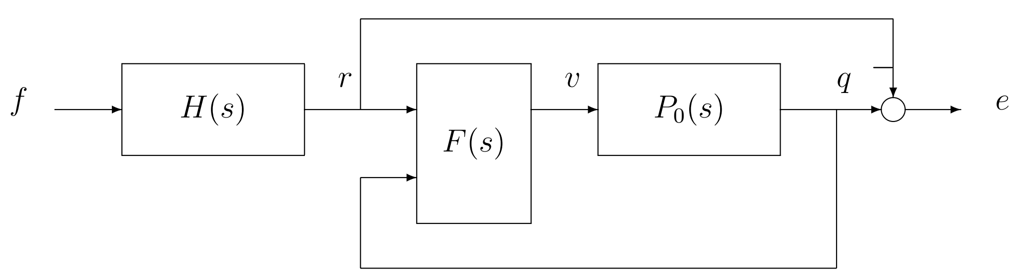

Figure 1.1: Design setup for Problem 1.1

[1] Version of February 4, 2004

(a) The feedback optimization problem formulated above is a special case of a standard LTI feedback optimization setup. Express the corresponding signals w, u, z, u in terms of f, r, v, q, e, and write down the resulting plant transfer matrix $P = P(s)$.

(b) Write down a (minimal) state space model for P.

(c) Find all frequencies $\omega \in \mathbf{R}$ at which the setup has control singularity or sensor singularity.

(d) Suggest a way to modify the setup, by introducing extra cost and disturbance variables, scaled by a single real parameter $d \in \mathbf{R}$, so that the parameterized problem becomes well-posed for $d \neq 0$, and the original ill-posed problem is recovered at $d = 0$.

(e) Write and test a MATLAB function, utilizing `h2syn.m`, which takes $d > 0$ as an input and produces the H2 optimal controller.

Problem 1.2

Consider the feedback design setup from Figure 1.2. It is frequently claimed that location

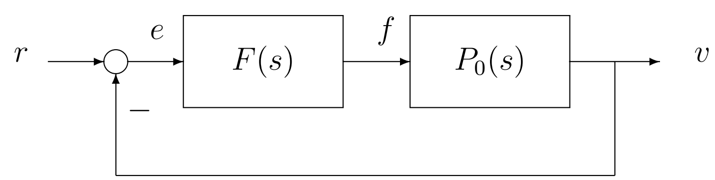

Figure 1.2: Design Setup For Problem 1.2

of unstable zeros of P_0 limits the maximal achievable closed loop bandwidth, which can be defined as the largest $\omega_0 > 0$ such that $|S(j\omega)| \leq 0.1$ for all $\omega \in [0, \omega_0]$, where

$$S = \frac{1}{1 + P_0 F}$$

is the closed loop sensitivity function. While mathematically this is not exactly true, the only way to achieve a sufficiently large bandwidth is by making $|S(j\omega)|$ extremely large at other frequencies.

You are asked to verify this using H-Infinity optimization on the following setup. Let

$$P_0(s) = \frac{s - a}{s + 1},$$

where $a > 0$ is a positive parameter (location of the unstable zero). For

$$H(s) = 10\frac{(s/c)^2 + \sqrt{2}s/c + 1}{(s/b)^2 + \sqrt{2}s/b + 1},$$

where b, c are positive parameters, and $c \gg b$, examine the possibility of finding a controller F which makes $|S(j\omega)H(j\omega)| < 1$ for *all* $\omega \in \mathbf{R}$. Since $|H(j\omega)| \approx 10$ for $\omega \ll b$, and $|H(j\omega)| \approx 10(b/c)^2 \ll 1$ for $\omega \gg c$, a controller satisfying condition $|S(j\omega)H(j\omega)| < 1$ will provide (at least) the closed loop bandwidth b.

For all $a \in \{0.1, 1, 10\}$, use H-Infinity optimization to find, with relative accuracy 20 percent, the maximal b such that the objective $|S(j\omega)H(j\omega)| < 1$ can be achieved with $c = 20b$. Make a conclusion about the relation between a and the achievable closed loop bandwidth.