Massachusetts Institute of Technology

Department of Electrical Engineering and Computer Science

6.245: MULTIVARIABLE CONTROL SYSTEMS

by A. Megretski

Problem Set 10 (due May 12, 2004) [1]

Problem 10.1

For a cone $\mathbf{\Delta} = \{\Delta\}$ of complex n-by-m matrices, and for a complex m-by-n matrix M, the quantity $\mu_{\mathbf{\Delta}}(M)$ is defined by

$$\mu_{\mathbf{\Delta}}(M) = (\inf\{\|\Delta\| : \ \Delta \in \mathbf{\Delta}, \ \det(I - M\Delta) = 0\})^{-1}$$

(in particular, $\mu_{\mathbf{\Delta}}(M) = 0$ is $I - M\Delta$ is invertible for all $\Delta \in \mathbf{\Delta}$). Such quantity, called *structured singular value* of M (where $\mathbf{\Delta}$ is what defines the "structure"), plays an important role in analysing robust stability.

When $\mathbf{\Delta}$ is the cone of *all* matrices, $\mu_{\mathbf{\Delta}}(M)$ equals the usual largest singular number of M. When $\mathbf{\Delta}$ is the set of all diagonal matrices with complex entries, $\mu_{\mathbf{\Delta}}(M) = \mu_{\mathbf{C}}(M)$ is called the *complex structured singular value*. When $\mathbf{\Delta}$ is the set of all diagonal matrices with real entries, $\mu_{\mathbf{\Delta}}(M) = \mu_{\mathbf{R}}(M)$ is called the *real structured singular value*.

Let $\mathbf{\Delta}$ be the cone of diagonal matrices with complex entries z_i such that $\text{Re}(z_i) \geq |\text{Im}(z_i)|$. Our objective is to produce a method for estimating $\mu_{\mathbf{\Delta}}(M)$, based on semidefinite programming.

(a) Describe the set of all quadratic constraints which are satisfied for the relation between two complex numbers w and v satisfying $w = zv$, where $\text{Re}(z) \geq |\text{Im}(z)|$.

(b) Use the result of (a) to develope an LMI optimization algorithm for calculating an upper bound $\hat{\mu}_{\mathbf{\Delta}}(M)$ of $\mu_{\mathbf{\Delta}}(M)$ for an arbitrary n-by-n complex matrix M.

[1] Version of May 6, 2004

(c) Test the upper boud on a set of randomply generated 3-by-3 and 10-by-10 complex matrices M. Compare $\hat{\mu}_{\mathbf{\Delta}}(M)$ with $\mu_{\mathbf{C}}(M)$, which can be estimated using MATLAB's

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bounds=mu(M)
```

(the two components of output `bounds` will be an upper and a lower bound of $\mu_{\mathbf{C}}(M)$).

Problem 10.2

In the design setup shown on Figure 10.1, r is the reference signal, y is measured plant output, u is control action, and $e = y - r$ is tracking error. Transfer functions W_1 (reference

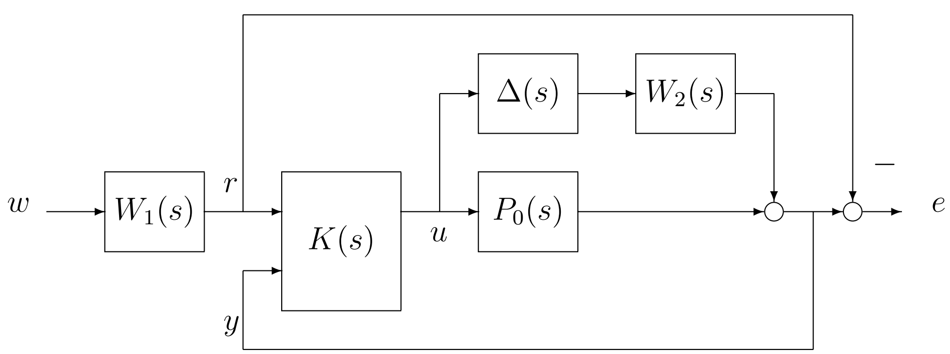

Figure 10.1: Design setup for Problem 10.2

signal shaping filter), P_0 (nominal plant model), and W_2 (uncertainty weight) are given:

$$W_1(s) = \frac{1}{1+20s},\ W_2(s) = r\frac{s+1}{s+10},\ P_0(s) = \frac{s-2}{s^2-1},$$

where $r > 0$ is a parameter. $\Delta = \Delta(s)$ is the normalized uncertainty, ranging over the set of all stable transfer functions with $\|\Delta\|_\infty \le 1$. The objective is to design an LTI controller $K = K(s)$ of order not larger than 8, which stabilizes the feedback system for all possible Δ ("robust stabilization"), while trying to make the worst (again, over all possible Δ) closed loop H-Infinity norm from w to e ("robust performance") as small as possible.

(a) Find the maximal value r_0 of those $r > 0$ for which robust stabilization is possible.

(b) For $r = 0.1r_0$, use D-K iterations of H-Infinity optimization and semidefinite programming to minimize robust performance.