Massachusetts Institute of Technology

Department of Electrical Engineering and Computer Science

6.243j (Fall 2003): DYNAMICS OF NONLINEAR SYSTEMS

by A. Megretski

Problem Set 7[1]

Problem 7.1

A stable linear system with a relay feedback excitation is modeled by

$$\dot{x}(t) = Ax(t) + B\text{sgn}(Cx(t)), \tag{7.1}$$

where A is a Hurwitz matrix, B is a column matrix, C is a row matrix, and $\text{sgn}(y)$ denotes the sign nonlinearity

$$\text{sgn}(y) = \begin{cases} 1, & y > 0, \\ 0, & y = 0, \\ -1, & y < 0. \end{cases}$$

For $T > 0$, a $2T$-periodic solution $x = x(t)$ of (7.1) is called a *regular unimodal limit cycle* if $Cx(t) = -Cx(t+T) > 0$ for all $t \in (0, T)$, and $CAx(0) > |CB|$.

(a) Derive a necessary and sufficient condition of exponential local stability of the regular unimodal limit cycle (assuming it exists and A, B, C, T are given).

(b) Use the result from (a) to find an example of system (7.1) with a Hurwitz matrix A and an *unstable* regular unimodal limit cycle.

Problem 7.2

A linear system controlled by modulation of its coefficients is modeled by

$$\dot{x}(t) = (A + Bu(t))x(t), \tag{7.2}$$

where A, B are fixed n-by-n matrices, and $u(t) \in \mathbf{R}$ is a scalar control.

[1] Posted November 5, 2003. Due date November 12, 2003

(a) Is it possible for the system to be controllable over the set of all non-zero vectors $\bar{x} \in \mathbf{R}^n$, $\bar{x} \neq 0$, when $n \geq 3$? In other words, is it possible to find matrices A, B with $n > 2$ such that for every non-zero $\bar{x}_0, \bar{x}_1$ there exist $T > 0$ and a bounded function $u : [0, T] \mapsto \mathbf{R}$ such that the solution of (7.2) with $x(0) = \bar{x}_0$ satisfies $x(T) = \bar{x}_1$?

(b) Is it possible for the system to be full state feedback linearizable in a neigborhood of some point $\bar{x}_0 \in \mathbf{R}^n$ for some $n > 2$?

Problem 7.3

A nonlinear ODE control model with control input u and controlled output y is defined by equations

$$\begin{aligned} \dot{x}_1 &= x_2 + x_3^2, \\ \dot{x}_2 &= (1 - 2x_3)u + a\sin(x_1) - x_2 + x_3 - x_3^2, \\ \dot{x}_3 &= u, \\ y &= x_1, \end{aligned}$$

where a is a real parameter.

(a) Output feedback linearize the system over a largest subset X_0 of $\mathbf{R}^3$.

(b) Design a (dynamical) feedback controller with inputs $x(t), r(t)$, where $r = r(t)$ is the reference input, such that for every bounded $r = r(t)$ the system state $x(t)$ stays bounded as $t \to \infty$, and $y(t) \to r(t)$ as $t \to \infty$ whenever $r = r(t)$ is constant.

(c) Find all values of $a \in \mathbf{R}$ for which the open loop system is full state feedback linearizable.

(d) Try to design a dynamical feedback controller with inputs $y(t), r(t)$ which achieves the objectives from (b). Test your design by a computer simulation.