Massachusetts Institute of Technology

Department of Electrical Engineering and Computer Science

6.243j (Fall 2003): DYNAMICS OF NONLINEAR SYSTEMS

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Problem Set 5[1]

Problem 5.1

$y(t) \equiv a$ is an equilibrium solution of the differential equation

$$y^{(3)}(t) + \ddot{y}(t) + \dot{y}(t) + 2\sin(y(t)) = 2\sin(a),$$

where $a \in \mathbf{R}$ and $y^{(3)}$ denotes the third derivative of y. For which values of $a \in \mathbf{R}$ is this equilibrium locally exponentially stable?

Problem 5.2

In order to solve a quadratic matrix equation $X^2 + AX + B = 0$, where A, B are given n-by-n matrices and X is an n-by-n matrix to be found, it is proposed to use an iterative scheme

$$X_{k+1} = X_k^2 + AX_k + X_k + B.$$

Assume that matrix X_* satisfies $X_*^2 + AX_* + B = 0$. What should be required of the eigenvalues of X_* and $A+X_*$ in order to guarantee that $X_k \to X_*$ exponentially as $k \to \infty$ when $\|X_0 - X_*\|$ is small enough? You are allowed to use the fact that matrix equation

$$ay + yb = 0,$$

where a, b, y are n-by-n matrices, has a non-zero solution y if and only if $\det(sI - a) = \det(sI + b)$ for some $s \in \mathbf{C}$.

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Problem 5.3

Use the Center manifold theory to prove local asymptotic stability of the equilibrium at the origin of the Lorentz system

$$\begin{cases} \dot{x} = -\beta x + yz, \\ \dot{y} = -\sigma y + \sigma z, \\ \dot{z} = -yx + \rho y - z, \end{cases}$$

where β, σ are positive parameters and $\rho = 1$. Estimate the rate of convergence of $x(t), y(t), z(t)$ to zero.

Problem 5.4

Check local asymptotic stability of the periodic trajectory $y(t) = \sin(t)$ of system

$$\ddot{y}(t) + \dot{y}(t) + y^3 = -\sin(t) + \cos(t) + \sin^3(t).$$

Problem 5.5

Find all values of parameter $a \in \mathbf{R}$ such that every solution $x : [0, \infty) \mapsto \mathbf{R}^2$ of the ODE

$$\dot{x}(t) = \epsilon \begin{bmatrix} \cos(2t) & a \\ \cos^4(t) & \sin^4(t) \end{bmatrix} x(t)$$

converges to zero as $t \to \infty$ when $\epsilon > 0$ is a sufficiently small constant.