Problem 21.2 Hints

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Step 1: Show that the problem reduces to

$$\inf_{\Delta \in \mathbb{R}^n} \left\{ \|\Delta\|_\infty \ : \ \begin{pmatrix} \Gamma_1' \\ \Gamma_2' \end{pmatrix} \Delta = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \right\}. \tag{1}$$

We have made this kind of argument several times.

Step 2: Develop a geometrical interpretation of this minimization, based on the intersection of sets

$$B_\beta = \left\{ \Delta \in \mathbb{R}^n \ : \ \|\Delta\|_\infty = \max_i |\delta_i| \le \beta \right\} \qquad \text{a } n\text{-d box}$$

and

$$L = \left\{ \Delta \in \mathbb{R}^n \ : \ \begin{pmatrix} \Gamma_1' \\ \Gamma_2' \end{pmatrix} \Delta = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \right\} \qquad \text{a } (n-2)\text{-d affine subspace.}$$

How large must β be for some Δ to be a member of both sets? How small *can* β be? Where in/on the box B_{β^*} is an "optimal" Δ going to appear (think about $\mathbb{R}^3$, where B_β is a cube and L is a line).

Step 3: We need to simplify the problem. Relax one dimension (say j) of the problem. That is, let

$$\Delta_{-j} = (\delta_1, \ldots, \delta_{j-1}, \delta_{j+1}, \ldots, \delta_n).$$

(omiting δ_j). Consider the relaxed problem (verify it is a relaxation?):

$$\inf \left\{ \beta_j \ : \ \|\Delta_{-j}\|_\infty \le \beta_j, \ \begin{pmatrix} \Gamma_1' \\ \Gamma_2' \end{pmatrix} \Delta = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \right\}.$$

What is the geometric interpretation here? There is a simple way to "remove" δ_j also from the linear constraint (by substitution). What remains is a problem that looks like our original rank-1 μ problem, but where all quantities are real. Use this formulation to derive a closed form for β_j. *Consider this the "crux" of the problem, and make sure to include it in your solution.* The answer should be in terms of the elements of Γ_1 and Γ_2.

Step 4: Recall this is a relaxed problem; when will the solution Δ correspond to a point in the box B_{β_j} (give a checkable condition)? In this case, then β_j is indeed the solution to (1). Argue that some dimension must produce a feasible solution. Which of the β_j must necessarily be the feasible one (that is, will it be large or small)?

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