6.231 Dynamic Programming

Midterm, Fall 2009

Problem 1 (30 points)

An enterprising financier dreams of making it big in the currency market. He may trade between n currencies $c_1, \ldots, c_n$ and can convert a unit of c_i to r_{ij} units of c_j, for any currency pair (c_i, c_j) (we assume $r_{ij} > 0$ for all i and j). He is looking for a cycle of currencies

$$c_{i_1} \to c_{i_2} \to \cdots \to c_{i_k} \to c_{i_1}$$

such that

$$r_{i_1 i_2} \cdot r_{i_2 i_3} \cdots r_{i_{k-1} i_k} \cdot r_{i_k i_1} > 1$$

(also known as an arbitrage opportunity).

(a) Formulate a shortest path problem, which has finite ($> -\infty$) shortest distances if and only if there is no arbitrage opportunity.

(b) Give an algorithm that detects the existence of an arbitrage opportunity.

Problem 2 (35 points)

Consider the basic problem over a finite horizon and assume that the system equation $x_{k+1} = f_k(x_k, u_k, w_k)$ has a special structure whereby from state x_k after applying u_k we move to an intermediate "post-decision state" $y_k = p_k(x_k, u_k)$ at cost $g_k(x_k, u_k)$. Then from y_k we move at no cost to the new state x_{k+1} according to

$$x_{k+1} = h_k(y_k, w_k),$$

where the distribution of the disturbance w_k depends only on y_k, and not on prior disturbances, states, and controls. Denote:

$J_k(x_k)$: The optimal cost-to-go starting at time k from state x_k.

$V_k(y_k)$: The optimal cost-to-go starting at time k from post-decision state y_k.

(a) Write a DP algorithm that generates only J_k, $k = 0, 1, \ldots, N-1$.

(b) Write a DP algorithm that simultaneously generates J_k and V_k, $k = 0, 1, \ldots, N-1$.

(c) Write a DP algorithm that generates only V_k, $k = 0, 1, \ldots, N-1$.

(d) Compare the algorithms and discuss the advantages and disadvantages of each in the case where J_k and or V_k are computed off-line, and the optimal policy is computed on-line with knowledge of J_k and/or V_k, $k = 0, 1, \ldots, N-1$.

Problem 3 (35 points)

A workshop manager has just bought an expensive new machine, and at each day he has two options: maintain the machine at cost M, or not maintain it. However, in the latter case, he runs the risk of a breakdown, which costs B and occurs with probability p_j, where j is the number of consecutive days after the preceding breakdown (if any) that the machine has not been maintained (e.g., on the first day with no maintenance the probability of breakdown is p_1, on the second successive day of no maintenance the probability of breakdown is p_2, etc). Assume that p_j is monotonically nondecreasing in j, and that there exists an integer m such that $p_m B > M$.

(a) Formulate this as an infinite horizon discounted cost problem with states $0, 1, \ldots, m$, and write the corresponding Bellman's equation.

(b) Characterize as best as you can the optimal policy.

(c) Formulate the infinite horizon average cost version of this problem with finite state space and write the corresponding Bellman's equation. State an assumption under which Bellman's equation holds.

6.231 Dynamic Programming and Stochastic Control
Fall 2015