Massachusetts Institute of Technology
Department of Electrical Engineering and Computer Science
6.111 - Introductory Digital Systems Laboratory

Problem Set 1 Solutions

Issued: February 8, 2006

Boolean Algebra Practice Problems *(not graded)*

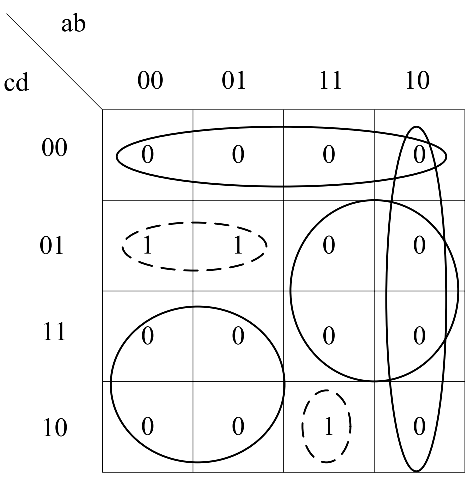
1) $a + 0 = a$
2) $\bar{a} \cdot 0 = 0$
3) $a + \bar{a} = 1$
4) $a + a = a$
5) $a + ab = a(1 + b) = a$
6) $a + \bar{a}b = (a + \bar{a})(a + b) = a + b$
7) $a(\bar{a} + b) = a\bar{a} + ab = ab$
8) $ab + \bar{a}b = b(a + \bar{a}) = b$
9) $(\bar{a} + \bar{b})(\bar{a} + b) = \bar{a}\bar{a} + \bar{a}b + \bar{b}\bar{a} + \bar{b}b = \bar{a} + \bar{a}b + \bar{a}\bar{b} = \bar{a}(1 + b + \bar{b}) = \bar{a}$
10) $a(a + b + c + ...) = aa + ab + ac + ... = a + ab + ac + ... = a$
11) $f(a, b, ab) = a + b + ab = a + b$
12) $f(a, b, \bar{a} \cdot \bar{b}) = a + b + \bar{a}\bar{b} = a + b + \bar{a} = 1$
13) $f[a, b, \overline{(ab)}] = a + b + \overline{(ab)} = a + b + \bar{a} + \bar{b} = 1$
14) $y + y\bar{y} = y$
15) $xy + x\bar{y} = x(y + \bar{y}) = x$
16) $\bar{x} + y\bar{x} = \bar{x}(1 + y) = \bar{x}$
17) $(w + \bar{x} + y + \bar{z})y = y$
18) $(x + \bar{y})(x + y) = x$
19) $w + [w + (wx)] = w$
20) $x[x + (xy)] = x$
21) $\overline{(\bar{x} + \bar{x})} = x$
22) $\overline{(x + \bar{x})} = 0$
23) $w + (w\bar{x}yz) = w(1 + \bar{x}yz) = w$
24) $\bar{w} \cdot \overline{(wxyz)} = \bar{w}(\bar{w} + \bar{x} + \bar{y} + \bar{z}) = \bar{w}$
25) $xz + \bar{x}y + zy = xz + \bar{x}y$
26) $(x + z)(\bar{x} + y)(z + y) = (x + z)(\bar{x} + y)$
27) $\bar{x} + \bar{y} + xy\bar{z} = \bar{x} + \bar{y} + \bar{z}$

Problem 1: Karnaugh Maps and Minimal Expressions

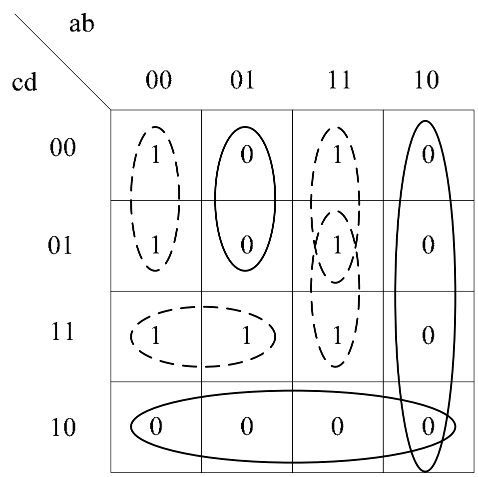
i) Truth Tables

abcd	1)	wxyz	2)
0000	0	0000	1
0001	1	0001	1
0010	0	0010	0
0011	0	0011	1
0100	0	0100	0
0101	1	0101	0
0110	0	0110	0
0111	0	0111	1
1000	0	1000	0
1001	0	1001	0
1010	0	1010	0
1011	0	1011	0
1100	0	1100	1
1101	0	1101	1
1110	1	1110	0
1111	0	1111	1

ii) Karnaugh Maps



(1)



(2)

iii) Minimum Sum of Products

(1) $\bar{a}\cdot\bar{c}\cdot d + a\cdot b\cdot c\cdot\bar{d}$

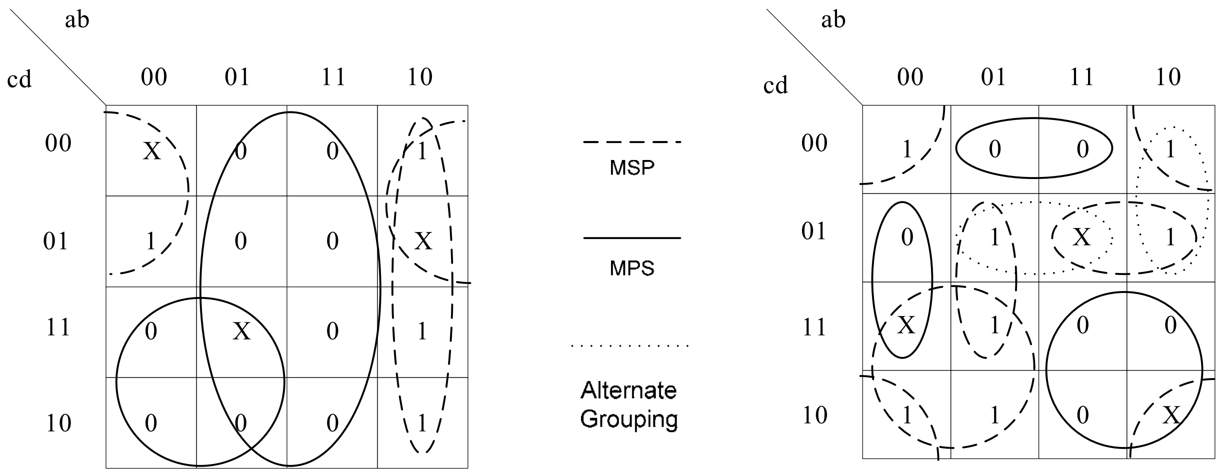
(2) $\bar{a}\cdot\bar{b}\cdot\bar{c} + a\cdot b\cdot\bar{c} + a\cdot b\cdot d + \bar{a}\cdot c\cdot d$

iv) Minimum Product of Sums

(1) $(c+d)(\bar{a}+b)(\bar{a}+\bar{d})(a+\bar{c})$

(2) $(\bar{c}+d)(\bar{a}+b)(a+\bar{b}+c)$

Problem 2: Karnaugh Maps with "Don't Cares"



(1) (2)

(1)

i. $\bar{b}\cdot\bar{c}+a\cdot\bar{b}$
ii. $\bar{b}\cdot(\bar{c}+a)$
iii. Both solutions are unique.
iv. Yes, MSP = MPS

(2)

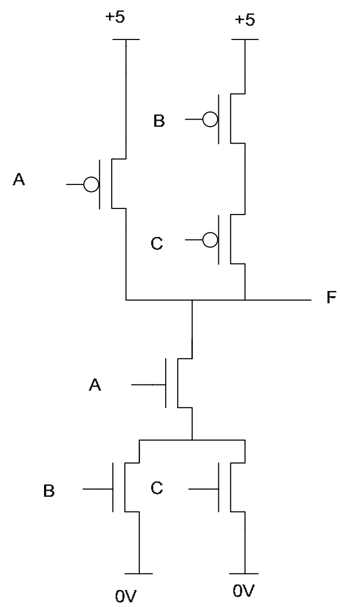
i. $\bar{b}\cdot\bar{d}+c\cdot\bar{a}+\bar{a}\cdot b\cdot d+a\cdot\bar{c}\cdot d$
ii. $(\bar{b}+c+d)(\bar{a}+\bar{c})(a+b+\bar{d})$
iii. The MPS solution is unique, the MSP is not.
In the MSP: $\bar{a}\cdot b\cdot d$ can be replaced with $\bar{c}\cdot b\cdot d$.
$a\cdot\bar{c}\cdot d$ can be replaced with $a\cdot\bar{c}\cdot\bar{b}$.
iv. No, MSP $\neq$ MPS.

Problem 3: DeMorgan's Theorem

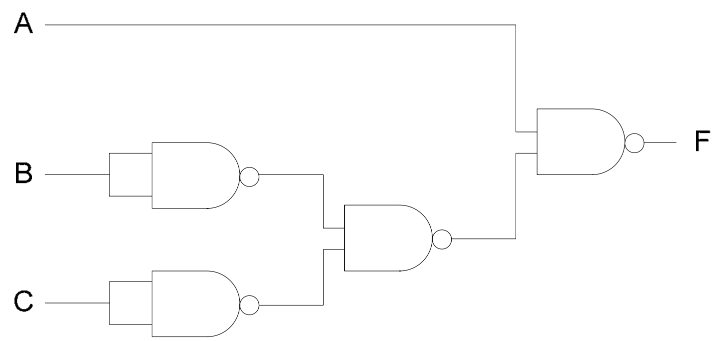
1) $\overline{(\bar{a}\cdot b\cdot\bar{c}\cdot\bar{d})}=a+\bar{b}+c+d$
2) $\overline{(\bar{a}+\bar{b}+c+\bar{d})}=a\cdot b\cdot\bar{c}\cdot d$
3) $(a\cdot\bar{d})\cdot(\bar{b}\cdot c)\cdot(c\cdot\bar{d})=a\cdot\bar{b}\cdot c\cdot\bar{d}$

Problem 4: Transistor/Gate Level Synthesis

1) Transistor implementation



2) NAND gate implementation



Problem 5: Setup and Hold times for D Flip-Flop

1) The setup time is twice the delay of the inverter, the hold time is zero.
2) The new memory element is a negative edge triggered flip-flop.
3) The setup time is $2t_{inv}$, the hold time is zero, and the clock to Q delay is $2t_{inv}$.