DEPARTMENT OF ELECTRICAL ENGINEERING AND COMPUTER SCIENCE
MASSACHUSETTS INSTITUTE OF TECHNOLOGY
CAMBRIDGE, MASSACHUSETTS 02139

DIFFERENTIATOR / INTEGRATOR INSIGHTS

For the differentiator:

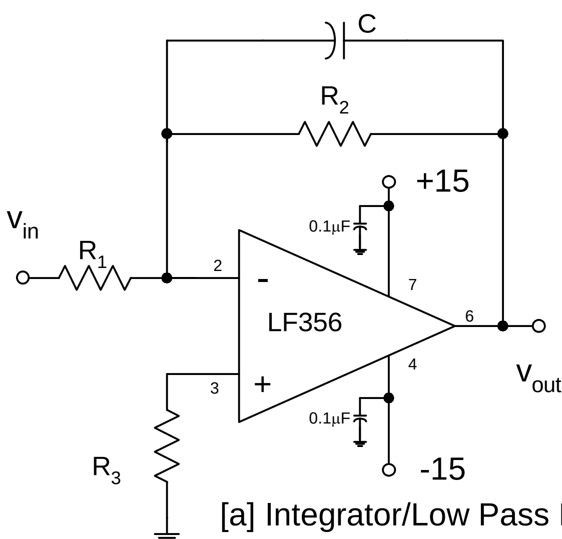
$$A_v = \frac{-R_2}{R_1 + \frac{1}{sC}} = \frac{-sCR_2}{sCR_1 + 1};\quad s = j\omega;$$ at frequencies 1/10 x f_{LO}, $sCR_1 << 1$, so

$$A_v = \frac{-sCR_2}{1}$$; multiplying by s equals differentiation.

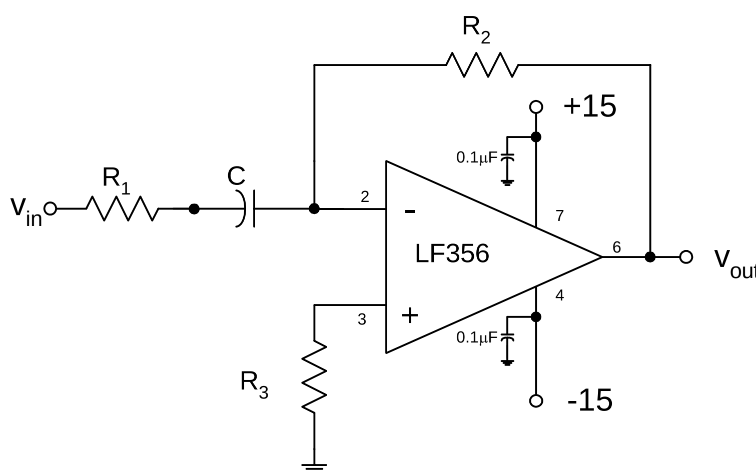
For the integrator:

$$A_v = -\frac{R_2/R_1}{1 + sCR_2};\quad s = j\omega;$$ at frequencies 10 x f_{HI}, $sCR_2 >> 1$, so

$$A_v = -\frac{R_2/R_1}{sCR_2} = -\frac{1}{sCR_1}$$; dividing by s equals integration.

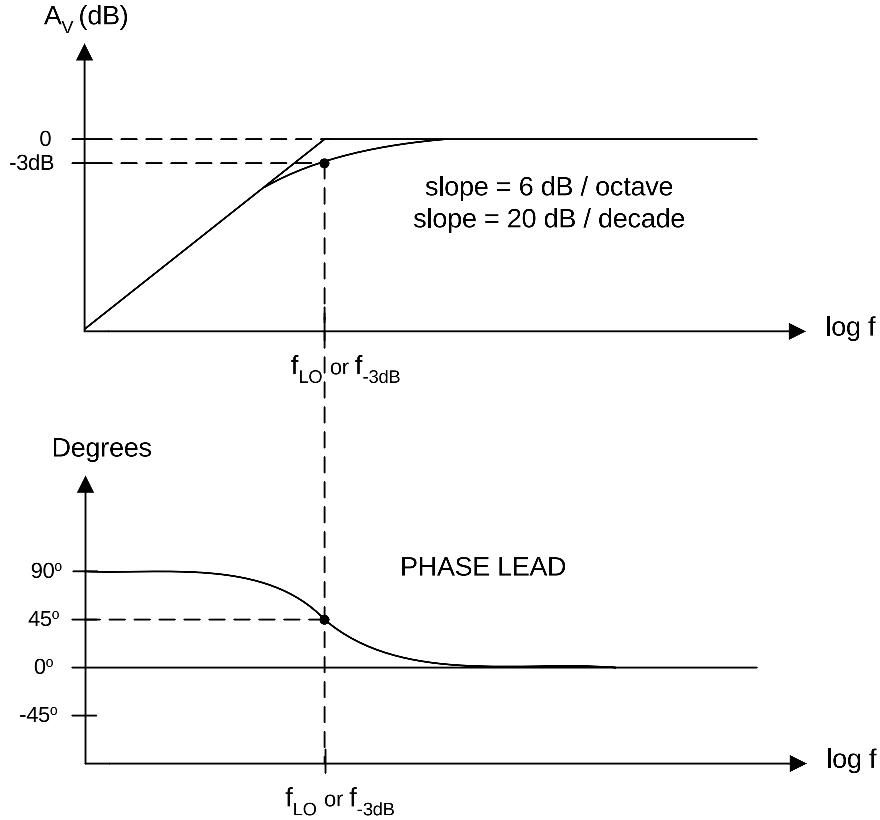


[a] Integrator/Low Pass Filter

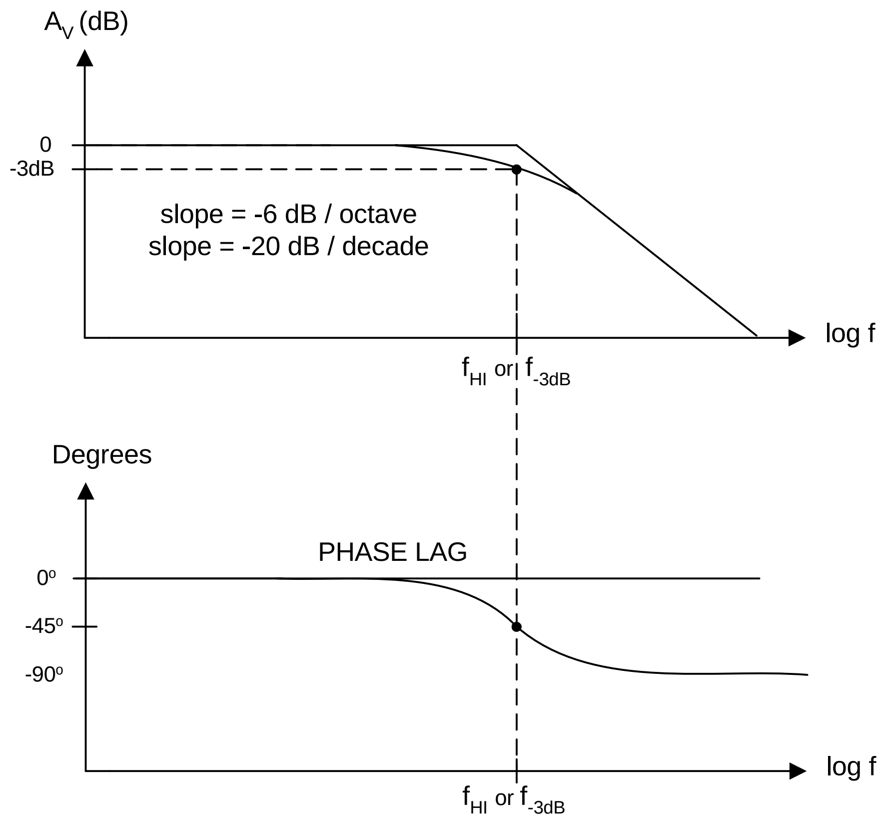


[b] Differentiator/High Pass Filter

Cite as: Ron Roscoe, course materials for 6.101 Introductory Analog Electronics Laboratory, Spring 2007. MIT OpenCourseWare (http://ocw.mit.edu/), Massachusetts Institute of Technology. Downloaded on [DD Month YYYY].



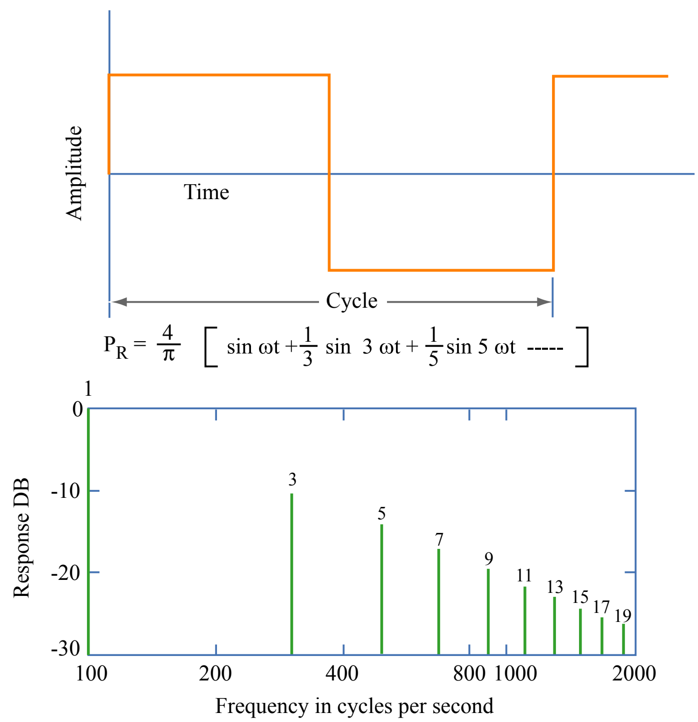
[c] High Pass Filter/Differentiator Bode plot: differentiation works only at $f \leq 1/10\ f_{LO}$



Cite as: Ron Roscoe, course materials for 6.101 Introductory Analog Electronics Laboratory, Spring 2007. MIT OpenCourseWare (http://ocw.mit.edu/), Massachusetts Institute of Technology. Downloaded on [DD Month YYYY].

[d] Low Pass Filter/Integrator Bode plot: integration works only at $f \geq 10\, f_{HI}$

It's easy to understand that an integrator will turn a square wave into a triangle wave and a differentiator will turn a triangle wave into a square wave because we all know the results of integrating or differentiating these simple functions. However, we get additional insight into how these circuits work their magic if we look at the amplitudes of the harmonics of these common waveforms:



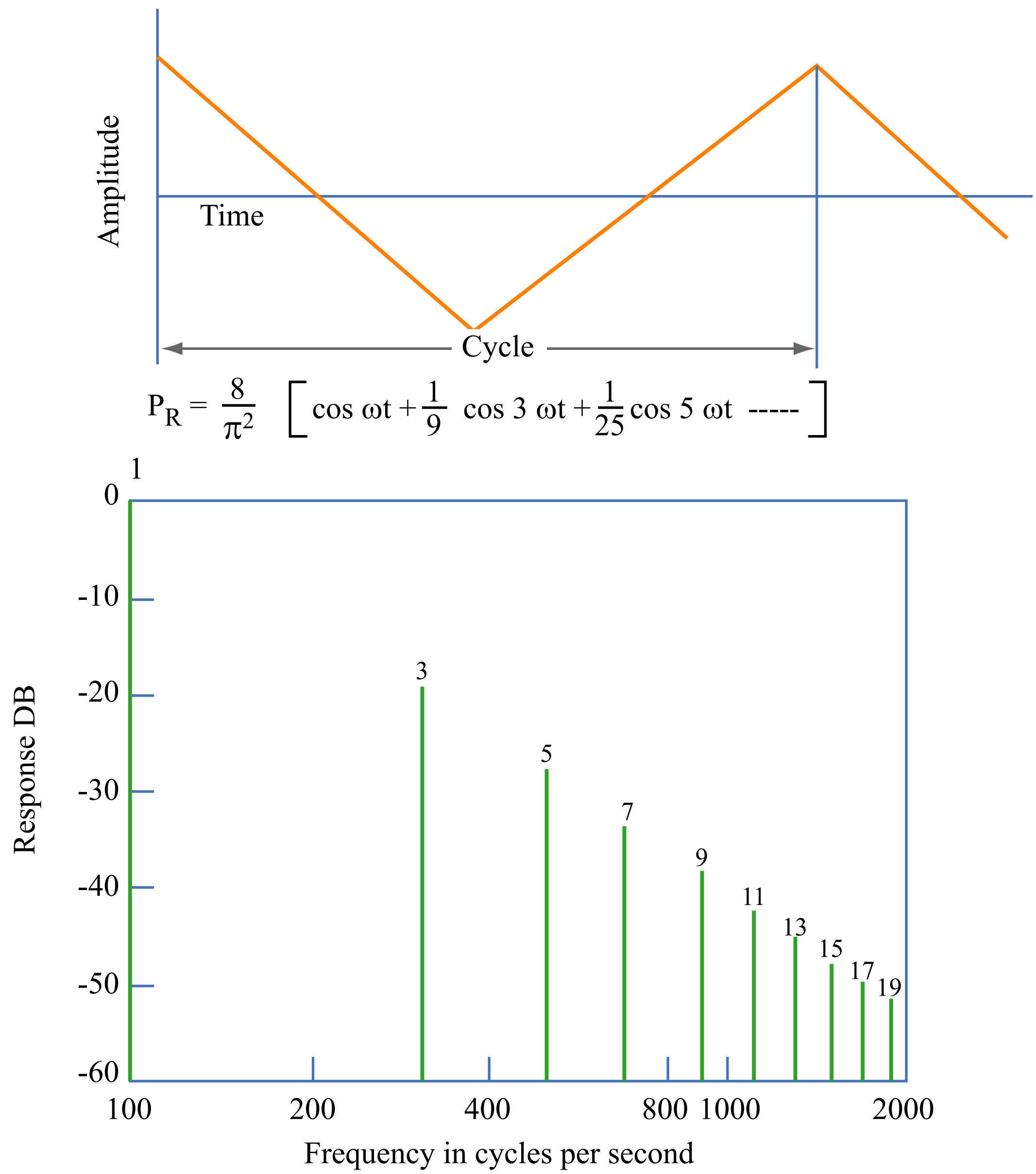
A rectangular wave, the equation of the wave, and the spectrum for a fundamental frequency of 100 cycles, that is, t = 1/100 second for a complete cycle.

Figure by MIT OpenCourseWare.

If we apply the above square wave to the input of an integrator, consider that the falling frequency response of the integrator above the corner frequency, f_{HI}, will attenuate the upper harmonics of the square wave relative to the lower harmonics and the fundamental. Since the only difference between a square wave and a triangle wave is the relative amplitudes of their harmonics, as well as phase shift, rolling off the harmonics of the square wave and phase shifting them creates a triangle wave. [Next page.]

Conversely, if we apply the triangle wave on the next page to the input of a differentiator, with the fundamental frequency at 1/10 of f_{LO}, the rising frequency response below the corner frequency f_{LO} will amplify the upper harmonics of the triangle wave relative to the fundamental and the lower harmonics and phase shift them, thus changing the triangle wave into a square wave.

Cite as: Ron Roscoe, course materials for 6.101 Introductory Analog Electronics Laboratory, Spring 2007. MIT OpenCourseWare (http://ocw.mit.edu/), Massachusetts Institute of Technology. Downloaded on [DD Month YYYY].



A triangular wave, the equation of the wave, and the spectrum for a fundamental frequency of 100 cycles, that is, t = 1/100 second for a complete cycle.

Figure by MIT OpenCourseWare.

Cite as: Ron Roscoe, course materials for 6.101 Introductory Analog Electronics Laboratory, Spring 2007. MIT OpenCourseWare (http://ocw.mit.edu/), Massachusetts Institute of Technology. Downloaded on [DD Month YYYY].