Homework 3 additional problems

1. *Reverse Jensen inequality.* Suppose f is convex, $\lambda_1 > 0$, $\lambda_i \leq 0$, $i = 2, \ldots, k$, and $\lambda_1 + \cdots + \lambda_n = 1$, and let $x_1, \ldots, x_n \in \textbf{dom}\, f$. Show that the inequality

$$f(\lambda_1 x_1 + \cdots + \lambda_n x_n) \geq \lambda_1 f(x_1) + \cdots + \lambda_n f(x_n)$$

always holds. *Hints.* Draw a picture for the $n = 2$ case first. For the general case, express x_1 as a convex combination of $\lambda_1 x_1 + \cdots + \lambda_n x_n$ and $x_2, \ldots, x_n$, and use Jensen's inequality.

2. *Reformulating constraints in* `cvx`*.* Each of the following `cvx` code fragments describes a convex constraint on the scalar variables `x`, `y`, and `z`, but violates the `cvx` rule set, and so is invalid. Briefly explain why each fragment is invalid. Then, rewrite each one in an equivalent form that conforms to the `cvx` rule set. In your reformulations, you can use linear equality and inequality constraints, and inequalities constructed using `cvx` functions. You can also introduce additional variables, or use LMIs. Be sure to explain (briefly) why your reformulation is equivalent to the original constraint, if it is not obvious.

Check your reformulations by creating a small problem that includes these constraints, and solving it using `cvx`. Your test problem doesn't have to be feasible; it's enough to verify that `cvx` processes your constraints without error.

Remark. This *looks* like a problem about 'how to use `cvx` software', or 'tricks for using `cvx`'. But it really checks whether you understand the various composition rules, convex analysis, and constraint reformulation rules.

(a) `norm([x + 2*y, x - y]) == 0`

(b) `square(square(x + y)) <= x - y`

(c) `1/x + 1/y <= 1; x >= 0; y >= 0`

(d) `norm([max(x,1), max(y,2)]) <= 3*x + y`

(e) `x*y >= 1; x >= 0; y >= 0`

(f) `(x + y)^2/sqrt(y) <= x - y + 5`

(g) `x^3 + y^3 <= 1; x >= 0; y >= 0`

(h) `x + z <= 1 + sqrt(x*y - z^2); x >= 0; y >= 0`

3. *Optimal activity levels.* Solve the optimal activity level problem described in exercise 4.17 in *Convex Optimization*, for the instance with problem data

$$A = \begin{bmatrix} 1 & 2 & 0 & 1 \\ 0 & 0 & 3 & 1 \\ 0 & 3 & 1 & 1 \\ 2 & 1 & 2 & 5 \\ 1 & 0 & 3 & 2 \end{bmatrix}, \quad c^{\mathrm{max}} = \begin{bmatrix} 100 \\ 100 \\ 100 \\ 100 \\ 100 \end{bmatrix}, \quad p = \begin{bmatrix} 3 \\ 2 \\ 7 \\ 6 \end{bmatrix}, \quad p^{\mathrm{disc}} = \begin{bmatrix} 2 \\ 1 \\ 4 \\ 2 \end{bmatrix}, \quad q = \begin{bmatrix} 4 \\ 10 \\ 5 \\ 10 \end{bmatrix}.$$

You can do this by forming the LP you found in your solution of exercise 4.17, or more directly, using `cvx`. Give the optimal activity levels, the revenue generated by each one, and the total revenue generated by the optimal solution. Also, give the average price per unit for each activity level, *i.e.*, the ratio of the revenue associated with an activity, to the activity level. (These numbers should be between the basic and discounted prices for each activity.) Give a *very brief* story explaining, or at least commenting on, the solution you find.