

Massachusetts Institute of Technology
Department of Electrical Engineering and Computer Science
6.061/6.690 Introduction to Power Systems

Quiz 1 Solutions

March 16, 2011

Statistics: Quiz Mean was 58
Standard Deviation was 23
Median Grade was 60

Problem 1: You will recognize this as a delta-wye connected transformer with a $+30^\circ$ phase shift from primary to secondary. Thus the voltages on the secondary side will be:

$$\begin{aligned}V_A &= \sqrt{3}NV e^{j\frac{\pi}{6}} \\V_B &= \sqrt{3}NV e^{-j\frac{\pi}{2}}\end{aligned}$$

The current in the resistor is:

$$I_R = \frac{V_A - V_B}{R} = \frac{3NV e^{j\frac{\pi}{3}}}{R}$$

Then currents on the primary of the transformer are:

$$\begin{aligned}I_a &= \frac{3N^2V}{R} e^{j\frac{\pi}{3}} \\I_b &= \frac{6N^2V}{R} e^{-j\frac{2\pi}{3}} \\I_c &= \frac{3N^2V}{R} e^{j\frac{\pi}{3}}\end{aligned}$$

Noting that:

$$\frac{3N^2V}{R} = \frac{3 \times 100 \times 100}{30,000} = 1$$

Then:

$$\begin{aligned}I_a &= 1e^{j\frac{\pi}{3}} \\I_b &= 2e^{-j\frac{2\pi}{3}} \\I_c &= 1e^{j\frac{\pi}{3}}\end{aligned}$$

The currents are sketched on the template. Note that the units are not related to the voltage units.

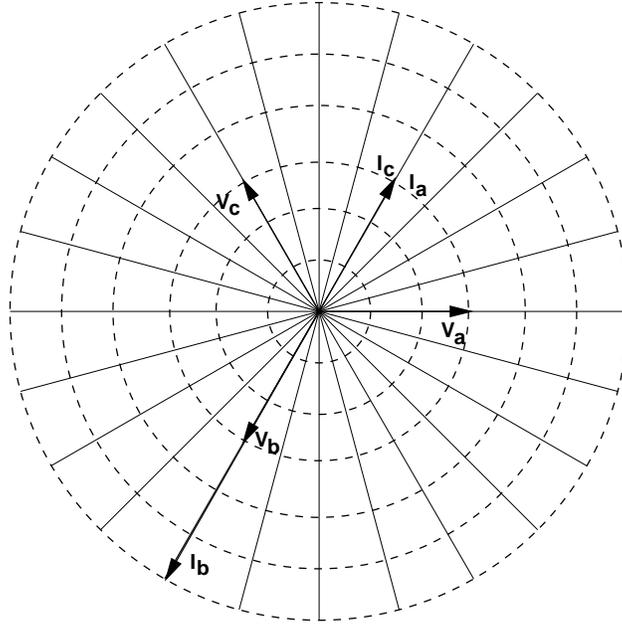


Figure 1: Problem 1 Currents

To find the real and reactive power in each phase, multiply by voltage: Note that:

$$e^{-j\frac{\pi}{3}} = \frac{1}{2} - j\frac{\sqrt{3}}{2}$$

$$(P + jQ)_A = 100 \times \left(\frac{1}{2} - j\frac{\sqrt{3}}{2} \right) = 50 - j50\sqrt{3} \approx 50 - j86.6$$

$$(P + jQ)_B = 100 \times 2 = 200$$

$$(P + jQ)_C = 100 \times \left(\frac{1}{2} + j\frac{\sqrt{3}}{2} \right) = 50 + j50\sqrt{3} \approx 50 + j86.6$$

Problem 2: Here, we can fall back on the simple relationships for the transmission line. If the voltage magnitudes are the same at both ends of the line. At the sending end:

$$P_s = \frac{V^2}{X} \sin \delta$$

$$Q_s = \frac{V^2}{X} (1 - \cos \delta)$$

And at the receiving end it is:

$$P_r = \frac{V^2}{X} \sin \delta$$

$$Q_r = -\frac{V^2}{X} (1 - \cos \delta)$$

Since $\delta = \frac{\pi}{6} = 30^\circ$, $\sin \delta = \frac{1}{2}$ and $\cos \delta = \frac{\sqrt{3}}{2}$. And $\frac{V^2}{X} = 10^6$. Then:

$$(P + jQ)_{\text{left}} = 500,000 + j10^6 \left(1 - \frac{\sqrt{3}}{2}\right) \approx 500,000 + j133,975$$

$$(P + jQ)_{\text{right}} = -500,000 + j133,975$$

To correct the power factor, we need

$$Q = \frac{V^2}{X_C} = 1 \times 10^6 \left(1 - \frac{\sqrt{3}}{2}\right)$$

Or, in this case:

$$X_c = \frac{1}{1 - \frac{\sqrt{3}}{2}} \approx 7.464 \Omega$$

Problem 3: The pulse launches a mode with $I_+ = \frac{V_+}{Z_0}$ and $V_s = V_+ + RI_+$, or $V_+ = \frac{V_s}{2}$. That propagates until it hits the shorted right-hand end of the line and is inverted. When that pulse gets back to the (matched) sending end, it generates no reflections. The resulting voltage in the middle of the line is shown in Figure 2

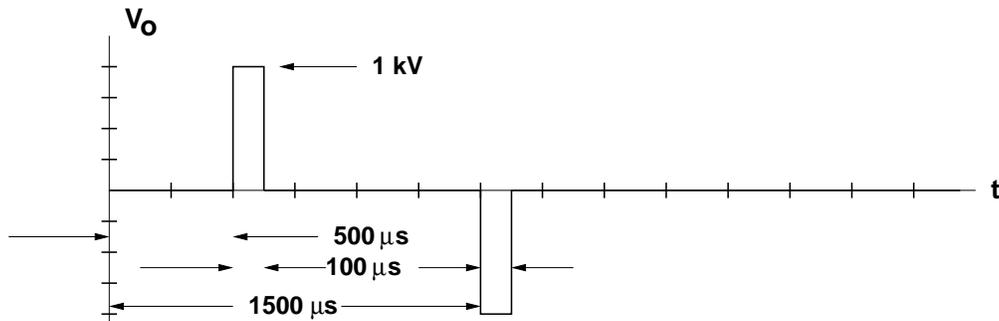


Figure 2: Problem 3 Solution

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