Massachusetts Institute of Technology

Department of Electrical Engineering and Computer Science

6.061/6.690 Introduction to Power Systems

Problem Set 11 Solutions April 25, 2011

Problem 1: Chapter 12, Problem 12 From the text, we have expressions for voltage: at the 'rectifier' end:

$$V_{DC} = \frac{3}{\pi} V_p \cos\alpha - \frac{3}{\pi} X I_{DC}$$

where V_p is the peak of line-line voltage: $V_p = \sqrt{6} V_{\ell\ell}$, if $V_{\ell\ell}$ is line-line, RMS voltage. Voltage drop across the 'fictitious' resistance is:

$$V_x = \frac{3}{\pi} X I_{DC} = \frac{3}{\pi} \times 1.5 \times 5,000 \approx 7162\text{V}$$

This can be used to calculate the firing angle α and the overlap angle u. At the rectifier end:

$$\begin{aligned}
\cos\alpha &= \frac{V_{DC} + V_x}{\frac{3}{\pi} V_p} \\
\cos(\alpha + u) &= \cos\alpha - \frac{2 X I_{DC}}{V_p}
\end{aligned}$$

This and the rest of the calculations are carried out by the script that is attached. Sending end numbers are:

```
Sending (Rectifier) end
Firing Angle = 23.9891 deg
Firing Angle Plus Overlap Angle = 28.1784 deg
Overlap Angle = 4.18929 deg
```

At the 'inverter' end of the line:

$$\begin{aligned}
\cos(\pi - \alpha) &= \frac{V_{DC} - V_X}{\frac{3}{\pi} V_p} \\
\cos(\alpha + u) &= \cos\alpha - \frac{2 X I_{DC}}{V_p}
\end{aligned}$$

```
Receiving (inverter) end
Firing Angle = 151.822 deg
Firing Angle Plus Overlap Angle = 156.011 deg
Overlap Angle = 4.18929 deg
```

To do the Fourier analysis, note that the AC side has alternating pulses of current with amplitude of 5,000 A and width of $\theta = 120°$. The Fourier series amplitude for harmonic of order n is:

$$I_n = I_{DC} \frac{4}{n\pi} \sin n\frac{\theta}{2} \sin n\frac{\pi}{2}$$

These evaluate to:

```
Time Harmonic Amplitudes for Six Pulse System
Harmonic Order    1  Current Amplitude = 5513.3
Harmonic Order    5  Current Amplitude = -1102.7
Harmonic Order    7  Current Amplitude = -787.6
Harmonic Order   11  Current Amplitude =  501.2
Harmonic Order   13  Current Amplitude =  424.1
Harmonic Order   17  Current Amplitude = -324.3
Harmonic Order   19  Current Amplitude = -290.2
Harmonic Order   23  Current Amplitude =  239.7
Harmonic Order   25  Current Amplitude =  220.5
```

Note the problem asks for only the first four of these, but I kept a few more to be consistent with the next part. A reconstructed time waveform is shown in Figure 1.

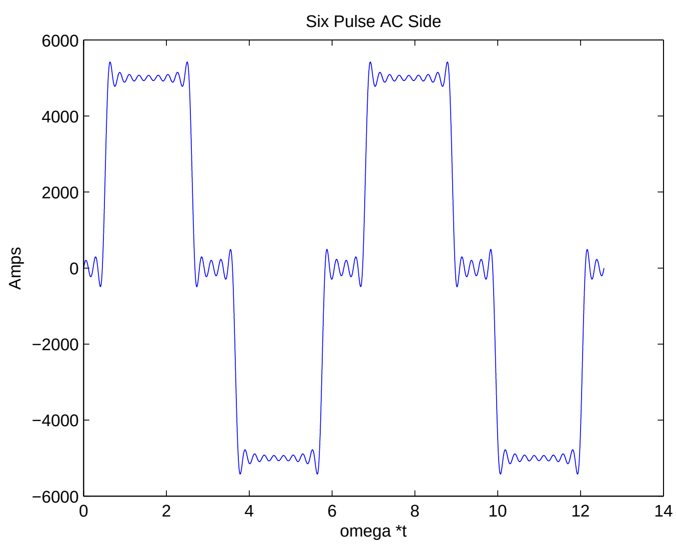

Figure 1: Reconstruction of Six Pulse Waveform, harmonics to order 25

For a twelve pulse system the amplitude of the harmonics of order 5, 7, 17 and 19 all cancel. The harmonics of the other orders remain the same. This needs a little explanation: we have not considered the use of transformers here, but to have the same AC and DC voltage levels, we would need transformers of 1/2 the ratio, so each of the two transformers would contribute

AC harmonics of 1/2 the amplitude as in the six pulse case, but these harmonics would add, restoring the amplitude to the same level. These would then be:

```
Harmonic Amplitudes for Twelve Pulse System
Harmonic Order    1  Current Amplitude = 5513.3
Harmonic Order   11  Current Amplitude =  501.2
Harmonic Order   13  Current Amplitude =  424.1
Harmonic Order   23  Current Amplitude =  239.7
Harmonic Order   25  Current Amplitude =  220.5
```

The reconstructed AC waveform is shown in Figure 2. It does look a little bit more sine wave like.

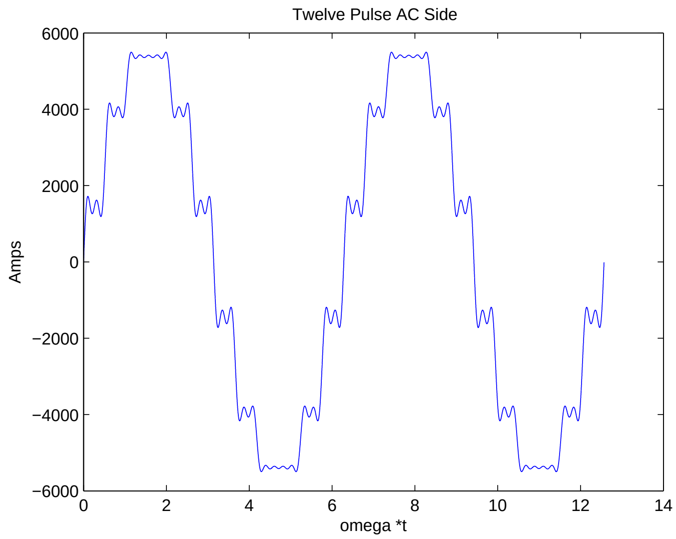

Figure 2: Reconstructed AC Waveform: Twelve Pulse

Script for Problem 12-12

```
% Problem 12-12

% basic parameters
X = 1.5;                % leakage at each end
Vl = 330e3;             % line-line voltage (AC)
Vdc = 400e3;            % DC voltage
I = 5e3;                % DC current

% first, get that mysterious overlap voltage

Vx = (3/pi)*X*I;
Vp = sqrt(2)*Vl;        % and this is the peak system voltage

alfs = acos((Vdc+Vx)/(3*Vp/pi));

upa = acos(cos(alfs)-2*X*I/Vp);

u = upa - alfs;

fprintf('Problem 12-12: Basic Analysis\n')
fprintf('Vx = %g\n', Vx)
fprintf('Sending (Rectifier) end\n')
fprintf('Firing Angle = %g deg\n', (180/pi)*alfs)
fprintf('Firing Angle Plus Overlap Angle = %g deg\n', (180/pi)*upa)
fprintf('Overlap Angle = %g deg\n', (180/pi)*u)

% other end
ppa = acos((Vdc- Vx)/((3/pi)*Vp));
alfr = pi - ppa;
apu = acos(cos(alfr) - 2*X*I/Vp);
ur = apu - alfr;

fprintf('Receiving (inverter) end\n')
fprintf('Firing Angle = %g deg\n', (180/pi)*alfr)
fprintf('Firing Angle Plus Overlap Angle = %g deg\n', (180/pi)*apu)
fprintf('Overlap Angle = %g deg\n', (180/pi)*ur)

% now do some Fourier Analysis

th = pi*2/3;              % this is the angle of each pulse

N = [1 5 7 11 13 17 19 23 25];
```

```
In = I * (4/pi) .* sin(N .* th/2) .* sin(N .* pi/2) ./ N;

fprintf('Time Harmonic Amplitudes for Six Pulse System\n')
for k = 1:length(N)
    fprintf('Harmonic Order %4.0f  Current Amplitude = %6.1f\n',N(k), In(k))
end

% now let's construct a figure of this

omt = 0:.001:4*pi;

Iac = zeros(size(omt));

for k = 1:length(N)
    Iac = Iac + In(k) .* sin (N(k) .* omt);
end

figure(1)
plot(omt, Iac)
title('Six Pulse AC Side')
ylabel('Amps')
xlabel('omega *t')

% now consider the 12-pulse situation
N = [1 11 13 23 25];

In = I * (4/pi) .* sin(N .* th/2) .* sin(N .* pi/2) ./ N;

fprintf('Harmonic Amplitudes for Twelve Pulse System\n')
for k = 1:length(N)
    fprintf('Harmonic Order %4.0f  Current Amplitude = %6.1f\n',N(k), In(k))
end

% now let's construct a figure of this

Iac = zeros(size(omt));

for k = 1:length(N)
    Iac = Iac + In(k) .* sin (N(k) .* omt);
end

figure(2)
plot(omt, Iac)
title('Twelve Pulse AC Side')
ylabel('Amps')
```

```
xlabel('omega *t')
```

Problem 2, part a: 14-2 With terminal voltage of 100 volts and 10 amperes flowing through $\frac{1}{2}\Omega$ internal voltage is:

$$G\Omega I_f = 95\text{V}$$

which means that

$$G = \frac{95}{180 \times 1} \approx 0.528\text{H}$$

And peak torque is

$$T = 10 \times 0.528 \approx 5.28\text{N-m}$$

Problem 2, Part b: 14-5 This is not as nasty a problem as it sounds. Note that we can easily calculate the motor constant: since $G\Omega I + RI = V$,

$$G = \frac{V - RI}{\Omega I}$$

And, since power is $P = G\Omega I^2$, we can find I for a given value of real power if we also know speed (which we do):

$$I^2 = \frac{P}{G\Omega}$$

So then to find speed vs. voltage, we do a cross plot: for the range of speed, we find power:

$$P = P_0 \left(\frac{\Omega}{\Omega_0}\right)^3$$

then calculate current according to the expression above, and then

$$V = (G\Omega + R)I$$

The result is plotted in Figure 3. Note a mistake in captioning (the figure identifies the wrong problem).

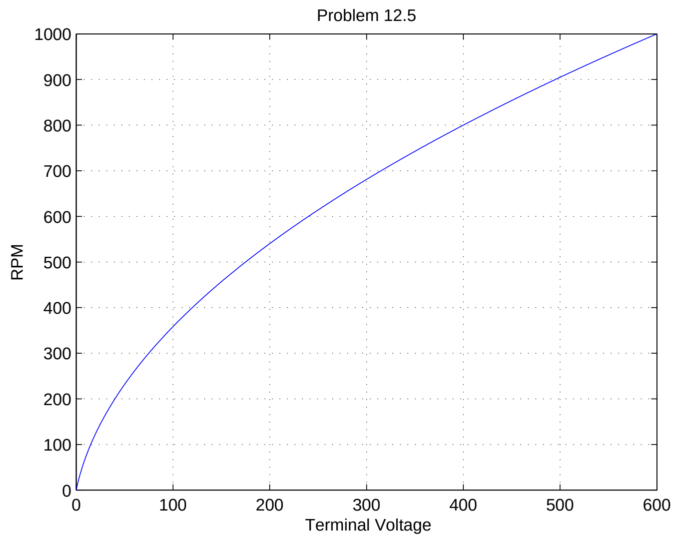

Figure 3: Speed vs. Voltage for Series Motor

Script for Problem 12-5

```
% Problem 12.5

Nz = 1000;
omz = (pi/30)*Nz;
Vz = 600;
Iz = 800;
Pz = 400e3;

R = 1/8;
G = (Vz-R*Iz)/(Iz*omz);

om = omz .* (0:.001:1);

P = Pz .* (om ./ omz) .^3;

I = sqrt(P ./ (G .* om));

V = (G .* om + R) .* I;
N = (30/pi) .* om;


figure(1)
plot(V, N);
title('Problem 12.5')
ylabel('RPM')
xlabel('Terminal Voltage')
grid on
```

Problem 3: 14-7 This is a piecewise linear situation that can be solved in each of three regions with the solutions patched together. In each region we have internal voltage:

$$
\begin{aligned}
E_a &= \tfrac{N}{N_0} R_0 I_F && 0 < I_F < 1 \\
E_a &= \tfrac{N}{N_0} \left(E_1 + R_1 \left(I_F - 1\right)\right) && 1 < I_F < 2 \\
E_a &= \tfrac{N}{N_0} \left(E_2 + R_2 \left(I_F - 1\right)\right) && 2 < I_F
\end{aligned}
$$

We can get minimum self-excitation speed by matching internal voltage with required voltage to make field current:

$$N = N_0 R_a + R_F R_0 \approx 450\text{RPM}$$

We could also write the second and third expressions as:

$$
\begin{aligned}
E_a &= \frac{N}{N_0} \left(V_1 + R_1 I_F\right) \\
E_a &= \frac{N}{N_0} \left(V_2 + R_2 I_F\right)
\end{aligned}
$$

Where, from the figure we have extracted the following data:

$$
\begin{aligned}
R_0 &= 200\Omega \\
R_1 &= 50\Omega \\
R_2 &= \frac{50}{3}\Omega \\
E_1 &= 200V \\
E_2 &= 250V \\
V_2 &= 150V \\
V_2 &= \frac{650}{3}V
\end{aligned}
$$

Note the equivalent circuit for the machine is shown in Figure 4.

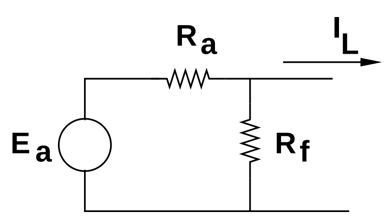

Figure 4: Equivalent Circuit

So that terminal voltage and field current are:

$$
\begin{aligned}
V &= E_a \frac{R_F}{R_F + R_a} - I_L \frac{R_F R_a}{R_F + R_a} \\
I_f &= \frac{V}{R_F}
\end{aligned}
$$

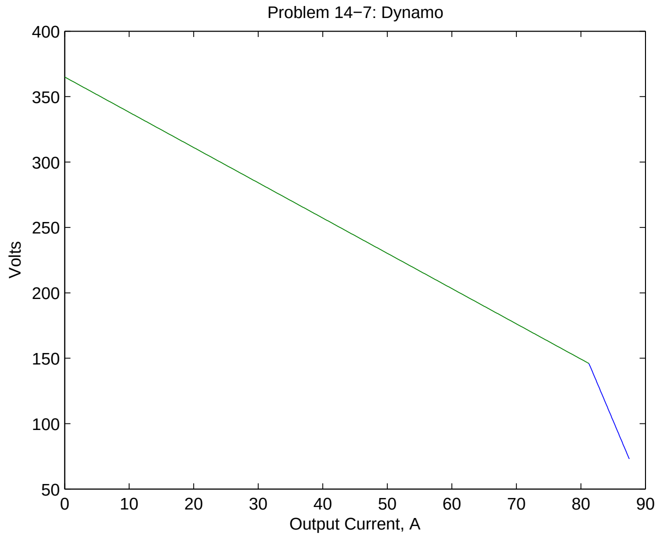

Figure 5: Voltage vs. Load Current

Now, we know the ranges of field current (1 A to 2 A and 2 A to the maximum, which is when:

$$I_{Fmax} = \frac{N}{N_0}\frac{V_2}{R_a + R_F - \frac{N}{N_0}R_2} = 5\text{A}$$

I is straightforward to get I_L in terms of I_F:

$$\begin{aligned} I_L &= \tfrac{N}{N_0}V_1R_a - I_F\tfrac{R_a+R_F-\frac{N}{N_0}R_1}{R_a} \qquad & 1 < I_F < 2 \\ I_L &= \tfrac{N}{N_0}V_2R_a - I_F\tfrac{R_a+R_F-\frac{N}{N_0}R_2}{R_a} \qquad & 2 < I_F < I_{Fmax} \end{aligned}$$

Then, for each of the two segments, first internal and then terminal voltage can be found:

$$\begin{aligned} E_{a1} &= \frac{N}{N_0}(V_1 + R_1I_{F1}) \\ E_{a2} &= \frac{N}{N_0}(V_2 + R_2I_{F2}) \end{aligned}$$

The plot of voltage with load current is shown in Figure 5.

To'flat compound', note that, with the addition of a series field:

$$\begin{aligned} V &= \frac{R_F}{R_F + R_a}E_a - \frac{R_aR_F}{R_a + R_F}I_L \\ E_a &= \frac{N}{N_0}(V_2 + R_2I_F) + \frac{N}{N_0}R_SI_L \end{aligned}$$

where R_S would be the characteristic of the series field. This suggests that terminal voltage V can be written out as:

$$V = \frac{R_F}{R_F + R_a} \left(\frac{N}{N_0} (V_2 + R_2 I_F) + \frac{N}{N_0} R_s I_L \right) - \frac{R_a R_F}{R_a + R_F} I_L$$

If the machine is indeed flat compounded so that V is constant, variations in I_F will not be of interest, so that what we need is for:

$$\frac{R_F}{R_F + R_a} \frac{N}{N_0} R_s = \frac{R_a R_F}{R_a + R_F}$$

We can accomplish this for only one speed, for which

$$R_s = \frac{N_0}{N} R_a$$

Now, the effective constant of a field winding is proportional to the number of turns, so if N_{ts} is the number of turns of the series field and N_{tf} is the number of turns of the shunt field, so that:

$$R_S = \frac{N_{ts}}{N_{tf}} R_2$$

Then required number of turns of the series field will be:

$$N_{ts} = N_{tf} \frac{R_S}{R_2} = N_{tf} \frac{N_0}{N} \frac{R_a}{R_2} = \frac{500}{1.25} \frac{2}{\frac{50}{3}} = 48 \text{turns}$$

Script for Problem 14-7

```
% Problem 14-7

Ra = 2;
Rf = 73;
E1 = 200;
E2 = 250;
R0 = 200;
R1 = 50;
R2 = 50/3;
V1 = E1 - R1;
V2 = E2 - 2*R2;
N0 = 1200;
N =1500;

% first break point is in speed
Ne = N0*(Ra+Rf)/R0;

fprintf('Excitation Speed = %g RPM\n', Ne)

I_f1 = 1:.01:2;
I_fmax = (N/N0)*V2 / (Ra+Rf-(N/N0)*R2);
V_oc = (N/N0)*(V2 + R2 * I_fmax);
fprintf('Open Circuit Voltage at %g RPM = %g\n', N, V_oc)


I_f2 = 2:.01:I_fmax;

I_L1 = (N/N0)*(V1/Ra) - I_f1 .* (Ra+Rf-(N/N0)*R1)/Ra;
I_L2 = (N/N0)*(V2/Ra) - I_f2 .* (Ra+Rf-(N/N0)*R2)/Ra;

figure(1)
plot(I_f1, I_L1, I_f2, I_L2)

Ea1 = (N/N0) .*(V1 + R1 .* I_f1);
Ea2 = (N/N0) .*(V2 + R2 .* I_f2);

Vt1 = Ea1 .* Rf/(Ra+Rf) - (Ra*Rf/(Ra+Rf)) .* I_L1;
Vt2 = Ea2 .* Rf/(Ra+Rf) - (Ra*Rf/(Ra+Rf)) .* I_L2;

figure(2)
plot(I_L1, Vt1, I_L2, Vt2)
```

Problem 4: Chapter 15, Problem 12 Peak torque is achieved when terminal current is exactly in quadrature with internal flux, in which case:

$$T = \frac{3}{2} p \lambda_0 I_0 = \frac{3}{2} \times 2 \times 0.4 \times 4 = 4.8 \text{ N-m}$$

With that torque, and noting that 4000 RPM = 418.9Radians/second,

$$P = \omega T = 418.9 \times 4.8 \approx 2011 \text{ Watts}$$

and with that condition, reactive voltage is in quadrature to internal voltage and terminal voltage is:

$$\begin{aligned} E_{\text{int}} &= 2 \times 418.9 \times .4 \approx 335.1\text{V (peak)} \\ V_x = 2 \times 418.9 \times .05 \times 4 \approx 167.6\text{V (peak)} & \\ V_{ph}^2 &= E_{\text{int}}^2 + V_x^2 \\ V_{\ell\ell} &= \sqrt{3} \times V_{ph} \approx 459\text{V (peak)} \end{aligned}$$

The machine can produce no torque when all terminal voltage is used to drive negative current in the d- axis to keep total current within rated:

$$\frac{V}{\omega} = L_0 (i_{sc} - i_{\text{max}})$$

Short circuit current is:

$$i_{sc} = \frac{\lambda_0}{L_0} = \frac{0.4}{.05} \approx 8\text{A}$$

so

$$\frac{V}{\omega} = .05 \times (8 - 4) = .2$$

or

$$\omega = \frac{V}{0.2} \approx 1,873.5$$

Or,

$$\omega_m = \frac{\omega}{p} \approx 936.75 \text{ radians/second} \approx 8945 \text{ RPM}$$

Problem 5: PWM The whole story is told by the script (which was the point of this problem: to write the script). the developed waveform is shown in Figure 6.

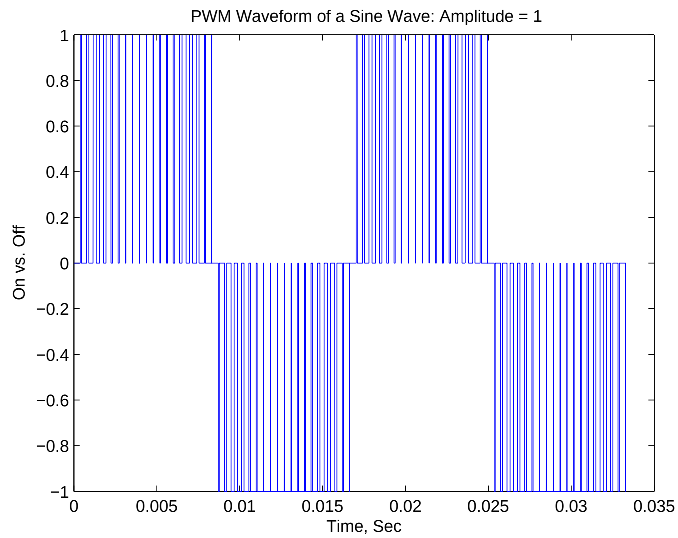

Figure 6: PWM Waveform

The fourier analysis of this is, for an amplitude of the fundamental of one (same as the triangle wave) is:

```
Fourier Analysis: fundemantal amplitude is 1
Harmonic   1 Amplitude = 0.999913
Harmonic   3 Amplitude = 0.000567261
Harmonic   5 Amplitude = 0.000557752
Harmonic   7 Amplitude = 0.000237784
Harmonic   9 Amplitude = 0.000238947
Harmonic  11 Amplitude = 9.1488e-05
Harmonic  13 Amplitude = -0.000103999
Harmonic  15 Amplitude = -0.000511369
Harmonic  17 Amplitude = -0.000412687
Harmonic  19 Amplitude = -0.000199636
Harmonic  21 Amplitude = -0.000310145
Harmonic  23 Amplitude = -0.000395562
Harmonic  25 Amplitude = -0.000803646
Harmonic  27 Amplitude = -0.000726942
Harmonic  29 Amplitude = -0.0013009
Harmonic  31 Amplitude = -0.00202079
Harmonic  33 Amplitude = -0.00553536
Harmonic  35 Amplitude = -0.0346489
```

For reduced amplitude, the harmonic content is:

```
Fourier Analysis: fundemantal amplitude is 0.25
Harmonic   1 Amplitude = 0.252131
Harmonic   3 Amplitude = 0.00103508
Harmonic   5 Amplitude = 0.00061986
Harmonic   7 Amplitude = 0.00038121
Harmonic   9 Amplitude = 0.000156627
Harmonic  11 Amplitude = 0.000260912
Harmonic  13 Amplitude = 3.41741e-06
Harmonic  15 Amplitude = 7.81786e-05
Harmonic  17 Amplitude = -0.000184951
Harmonic  19 Amplitude = 0.000229312
Harmonic  21 Amplitude = -0.000290529
Harmonic  23 Amplitude = 0.000140973
Harmonic  25 Amplitude = -0.000277316
Harmonic  27 Amplitude = -0.000161898
Harmonic  29 Amplitude = -0.00070203
Harmonic  31 Amplitude = -0.000270523
Harmonic  33 Amplitude = -0.00114894
Harmonic  35 Amplitude = -0.00167895
```

Script for Problem 5

```
f = 60;                 % basic electrical frequency
fp = 2400;              % PWM frequency
T = 1/30;               % do it for two cycles
d = 1e-6;               % 1 microsecond increments

[t, wp] = triangle(fp, T, d); %Here is your basic triangle waveform
wm = -wp;                     % positive and negative halves

s = sin(2*pi*f .* t);         % sine wave of fundamental frequency

pp = s > wp;                  % positive half test
pm = s < wm;                  % negative half test

pwm = pp - pm;                % this should be the whole thing

figure(2)
clf
plot(t, pwm)
title('PWM Waveform of a Sine Wave: Amplitude = 1')
ylabel('On vs. Off')
xlabel('Time, Sec')

% ok: now do a little fourier analysis
Nh = 1:2:35;

fprintf('Fourier Analysis: fundemantal amplitude is 1\n')

for k = 1:length(Nh)
    sn = sin(2*pi*Nh(k)*f .*t);
    fn = 2*sum(sn .* pwm) / length(t);
    fprintf('Harmonic %3.0f Amplitude = %g\n', Nh(k), fn);
end

s = .25*sin(2*pi*f .* t);         % sine wave of fundamental frequency

pp = s > wp;                  % positive half test
pm = s < wm;                  % negative half test

pwm = pp - pm;                % this should be the whole thing

fprintf('Fourier Analysis: fundemantal amplitude is 0.25\n')
```

```
for k = 1:length(Nh)
    sn = sin(2*pi*Nh(k)*f .*t);
    fn = 2*sum(sn .* pwm) / length(t);
    fprintf('Harmonic %3.0f Amplitude = %g\n', Nh(k), fn);
end
figure(3)
clf
plot(t, pwm)

figure(4)
clf
plot(t, s, t, wm)
----------------
function [t, w] = triangle(f, T, d)
% generates a triangle wave of amplitude 1
% frequency f
% length T
% increment d

T_c = 1/f;                  % length of one cycle
Ni = floor(.5*T_c/d);           % number of increments per half cycle
Nc = floor(T*f);           % number of cycles
%pause

% build first cycle

ws = (0:Ni-1) ./ (Ni-1);       % first half cycle
wt = (Ni-1:-1:0) ./ (Ni-1);    % second half cycle
wc = [ws wt];                  % first full cycle

w = wc;                        % start concatenating

for k = 1:Nc-1,
    w = [w wc];
end                            % all done

t = 0:d:d*(length(w)-1);           % so t is the same length as w
```