Massachusetts Institute of Technology
Department of Electrical Engineering and Computer Science
6.061 Introduction to Power Systems

Problem Set 5 Solutions February 27, 2011

Problem 1: The inductance and phase velocity are:

$$L = CZ_0^2 = 2 \times 10^{-10} \times 900 \approx -.18\mu\text{Hy/m}$$
$$c = \sqrt{\frac{1}{LC}} = \sqrt{\frac{1}{2 \times 10^{-10} \times 1.8 \times 10^{-7}}} \approx 1.66667 \times 10^8\text{m/S}$$

Before the load is connected, voltage is uniform and equal to the source and current is zero. So:

$$V_+ = \frac{V}{2}$$
$$V_- = \frac{V}{2}$$
$$I_+ = \frac{V}{2Z_0}$$
$$I_- = -\frac{V}{2Z_0}$$

When the switch is thrown, at the load end, V_- becomes zero, since the load end is matched to the line. For the first interval, the situation is as shown in Figure 1. The voltage at the matched end is V_+. When the reverse going pulse gets to the sending end, V_+ becomes equal to the source, since V_- is equal to zero. That takes another period of time to propagate to the receiving end, at which point the voltabe is equal to sending end voltage. This is shown in Figure 2. Transit time is:

$$T_t = \frac{L}{u} = \frac{100,000}{1.6667 \times 10^8} \approx 600\mu S$$

And it takes $2T_t$ for the transient to finish.

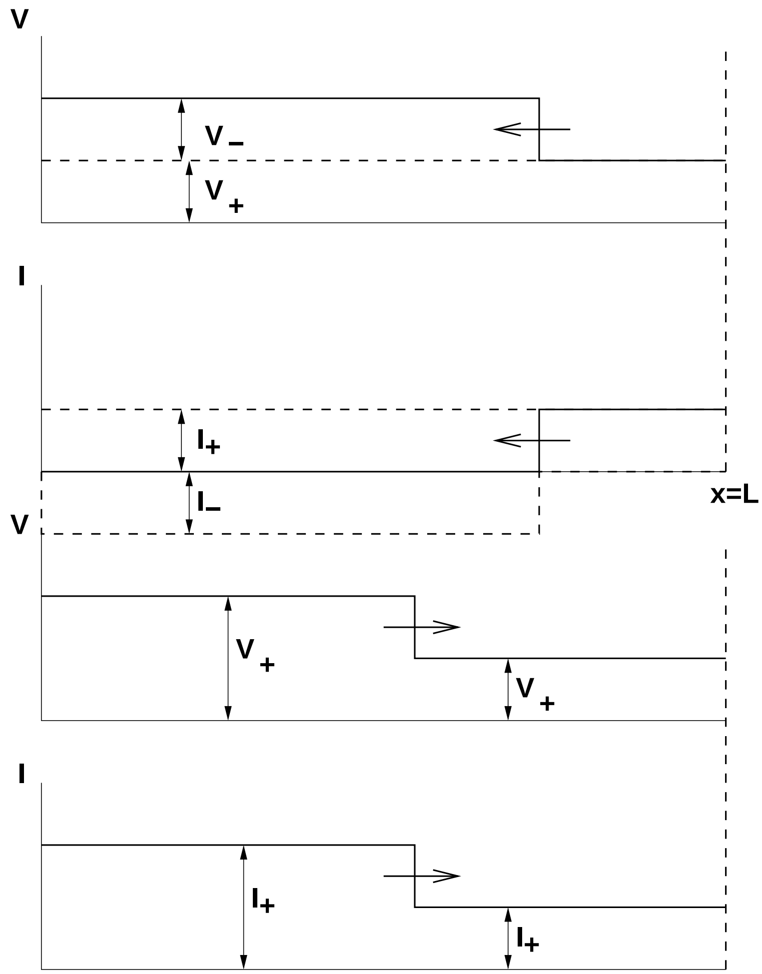

Figure 1: Pulses along the line

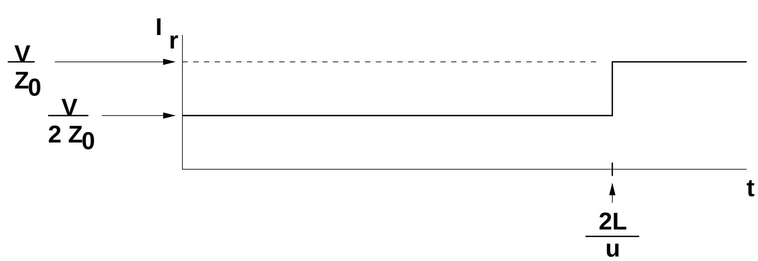

Figure 2: Termination Voltage

Problem 2: Voltage at the receiving end and impedance at the sending end are:

$$
\begin{aligned}
V_r &= V_s \frac{Z_t}{Z_t \cos k\ell + jZ_0 \sin k\ell} \\
Z_s &= Z_0 \frac{Z_t \cos k\ell + jZ_0 \sin k\ell}{jZ_t \sin k\ell + Z_0 \cos k\ell}
\end{aligned}
$$

If the line is open at the receiving end,

$$
\begin{aligned}
V_r &= \frac{V_S}{\cos k\ell} \\
Z_s &= = jZ_t \coth k\ell
\end{aligned}
$$

The rest of the calculations are carried out by Matlab (source appended) and the results are:

```
Line Open at Receiving End
Receiving end voltage =     46176.3 V
Sending End Current =       345.2 A
Line loaded with 60 Ohms
Receiving End Voltage =     45873.6 V
Sending end Current =       820.2 A
Receiving end Power = 35073062.2 W
Sending End Real Power = 35073062.2 W
Sending End Reactive Power = -11498241.4 VAR
```

Problem 3: (Chapter 6, Problem 6 from text)

Turns ratio is $N = \frac{13{,}800}{\sqrt{3}\times 480} \approx \frac{7967.4}{480} \approx 16.60$.

If the primary voltages are:

$$\begin{aligned} V_B &= Ve^{-j\frac{2\pi}{3}} \\ V_C &= Ve^{j\frac{2\pi}{3}} \end{aligned}$$

Then, on the secondary side,with respect to Figure 3:

$$\begin{aligned} V_{bc} &= \frac{V}{N}e^{-j\frac{2\pi}{3}} \\ V_{ca} &= \frac{V}{N}e^{j\frac{2\pi}{3}} \\ V_{ab} &= -V_{ca} - V_{bc} = -\frac{V}{N} \end{aligned}$$

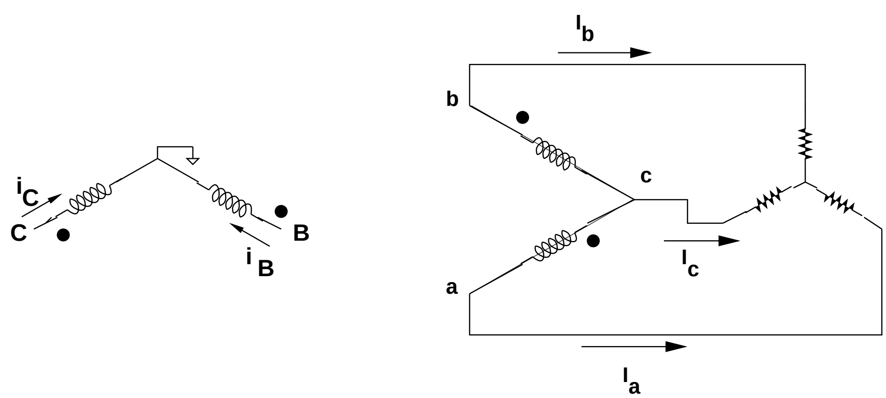

Figure 3: Equivalent Circuit with Currents

The actual phase voltages are shown in Figure 4

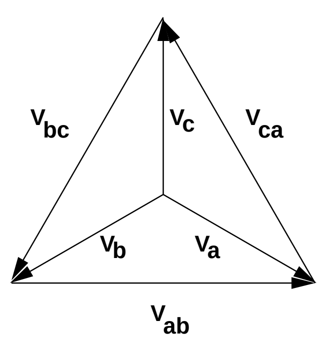

Figure 4: Secondary Side Voltages

The phase voltages are:

$$
\begin{aligned}
V_a &= \frac{V}{\sqrt{3}N} e^{-j\frac{\pi}{6}} \\
V_b &= \frac{V}{\sqrt{3}N} e^{j\frac{5\pi}{6}} \\
V_c &= \frac{V}{\sqrt{3}N} e^{-j\frac{\pi}{2}}
\end{aligned}
$$

If the loads are unity power factor and of magnitude I_0,

$$
\begin{aligned}
I_a &= I_0 e^{-j\frac{\pi}{6}} \\
I_b &= I_0 e^{-j\frac{5\pi}{6}} \\
I_c &= I + 0 e^{j\frac{\pi}{2}}
\end{aligned}
$$

On the wye side of the transformer:

$$
\begin{aligned}
I_A &= 0 \\
I_B &= \frac{I_b}{N} = \frac{I_0}{N} e^{-j\frac{5\pi}{6}} \\
I_C &= -\frac{I_a}{N} = \frac{I_0}{N} e^{j\frac{5\pi}{6}}
\end{aligned}
$$

This is shown in Figure 5

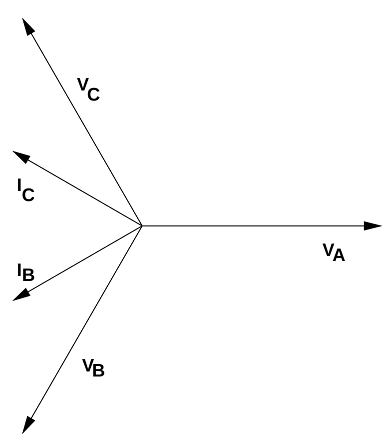

Figure 5: Primary Side Currents

As a test, we can see if the real and reactive powers are the same on the primary and secondary sides. On the primary side:

$$
\begin{aligned}
P_A &= 0 \\
P_B &= \frac{VI_0}{N} e^{-j\frac{\pi}{6}}
\end{aligned}
$$

$$\begin{aligned} P_C &= \frac{VI_0}{N}e^{j\frac{\pi}{6}} \\ P_A + P_B + P_C &= \frac{\sqrt{3}VI_0}{N} \end{aligned}$$

On the secondary side, power is:

$$P = 3\frac{VI_0}{\sqrt{3}N} = \frac{\sqrt{3}VI_0}{N}$$

A few numbers are:

```
Problem 6.6
Turns Ratio = 16.5988
Primary Power = 83138.4 + j 0
Secondary Powr = 83138.4 + j 2.01948e-28
```

Finally, if the ground is not connected on the primary side, the thing becomes a singla phase circuit, with $V_B - B_C = -j\sqrt{3}V$ Then: $V_{BC} = -V_{CA} = j\frac{1}{s}qrt3V2N$ And this drives a resistance of $R_{eq} = \frac{3}{2R}$.

Problem 4: Essentially the whole story is contained in the attached script. The first part is straightforward. The second part is done by first doing the delta-wye equivalent, finding the currents in the delta, re-assembling them into the wye and then transforming them across the transformer. The third part is a little tricky, but note that the voltages can be found by:

$$\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \begin{bmatrix} R_a + R_n & R_n & R_n \\ R_n & R_b + R_n & R_n \\ R_n & R_n & R_c + R_n \end{bmatrix} \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix}$$

This is easily inverted in Matlab to get the currents. A summary of what is calculated is:

```
Turns Ratio = 5.05181
Line-Neutral Voltage = 2424.87
Phase C current = 13.1966
Phase C real power = 32000
Phase A real power = 8000
Low side power = 48000
High side power = 48000
High side reactive = 13856.4
>> open p4.m
>> p6_6
Problem 6.6
Turns Ratio = 16.5988
Primary Power = 83138.4 + j 0
Secondary Powr = 83138.4 + j 2.01948e-28
>> p6_7
Part A: Secondary Side Grounded
Secondary Currents
Ia = 5.54256 + j 0 = 5.54256 angle 0
Ib = -4.6188 + j -8 = 9.2376 angle -2.0944
Ic = -4.6188 + j 8 = 9.2376 angle 2.0944
Primary Currents
IA = 0.676923 + j -0.532939 = 0.861538 angle -0.666946
IB = -0.676923 + j -0.532939 = 0.861538 angle -2.47465
IC = 2.9584e-16 + j 1.06588 = 1.06588 angle 1.5708
Check: Primary P = 6656 Q = 0 Secondary P = 6656 Q = 0
Part B: Secondary Side Ungrounded
Rab = 130  Rbc = 78  Rca = 130
Secondary Currents
Ia = 6.39526 + j -6.66134e-16 = 6.39526 angle -1.0416e-16
Ib = -3.19763 + j -8 = 8.61538 angle -1.95105
Ic = -3.19763 + j 8 = 8.61538 angle 1.95105
Primary Currents
IA = 0.639053 + j -0.532939 = 0.832113 angle -0.695102
IB = -0.639053 + j -0.532939 = 0.832113 angle -2.44649
```

```
IC = 2.9584e-16 + j 1.06588 = 1.06588 angle 1.5708
Check: Primary P = 6498 Q = 0 Secondary P = 6498 Q = 0
Part C: secondary grounded through a resistor
Secondary Currents
Ia = 5.93846 + j -2.29327e-16 = 5.93846 angle -3.86172e-17
Ib = -3.95897 + j -8 = 8.926 angle -2.03033
Ic = -3.95897 + j 8 = 8.926 angle 2.03033
Primary Currents
IA = 0.659341 + j -0.532939 = 0.847793 angle -0.679776
IB = -0.659341 + j -0.532939 = 0.847793 angle -2.46182
IC = 2.9584e-16 + j 1.06588 = 1.06588 angle 1.5708
Check: Primary P = 6583 Q = 0 Secondary P = 6583 Q = 0
```

Note that Matlab sometimes gives an odd rendition of zero (as in something times 10^-17,

Problem 5 Problem 7-10 from the text

$$
\begin{aligned}
R_1 &= .78 \times .03 \approx 0.0234 \\
R_2 &= 1 \\
R_3 &= \sqrt{2} \approx 1.4142 \\
R_4 &= 1
\end{aligned}
$$

So

$$
\mathrm{GMD} = \sqrt[4]{.0234 \times \sqrt{2}} \approx .4265\mathrm{m}
$$

Scripts

```
% 6.061 Problem Set 5 Problem 2 2/26/11

L = 1e5;          % line length 100 km
Z0 = 30;          % characteristic impedance
Cap = 2e-10;      % capacitance
om = 120*pi;      % frequency
Vs = 45000;       % sending end voltage
Ind = Cap * Z0^2; % line inductance
u = 1/sqrt(Ind*Cap);  % phase velocity
k = om/u;             % wavenumber

%part 1: line open
Vr = Vs / cos(k*L);     % receiving end voltage
Is = j*tan(k*L)*Vs/Z0;  % sending end voltage

fprintf('Line Open at Receiving End\n')
fprintf('Receiving end voltage = %10.1f V\n', Vr)
fprintf('Sending End Current = %10.1f A\n', abs(Is))

% part 2: resistive loading at 60 ohms
Zr = 60;
Vr = Vs * Zr/(Zr * cos(k*L) + j*Z0 * sin(k*L));
Zs = Z0 * (Zr*cos(k*L) + j*Z0*sin(k*L))/(j*Zr*sin(k*L) + Z0 * cos(k*L));
Is = Vs/Zs;

Ss = Vs*conj(Is);
Ps = real(Ss);
Qs = imag(Ss);
Pr = abs(Vr)^2/Zr;

fprintf('Line loaded with 60 Ohms\n')
fprintf('Receiving End Voltage = %10.1f V\n', abs(Vr))
fprintf('Sending end Current = %10.1f A\n', abs(Is))
fprintf('Receiving end Power = %10.1f W\n', Pr)
fprintf('Sending End Real Power = %10.1f W\n', Ps)
fprintf('Sending End Reactive Power = %10.1f VAR\n', Qs)
-----------------
% chapter 6, problem 6

V_0 = 13800/sqrt(3);
N = V_0/480;
V_A = V_0;
V_B = V_0 * exp(-j*2*pi/3);
V_C = V_0 * exp(j*2*pi/3);
```

```
V_s = 480/sqrt(3);
V_a = V_s * exp(-j*pi/6);
V_b = V_s * exp(-j*5*pi/6);
V_c = V_s * exp(j*pi/2);
I_a = 100 * exp(-j*pi/6);
I_b = 100 * exp(-j*5*pi/6);
I_c = 100 * exp(j*pi/2);
S_s = V_a * conj(I_a) + V_b * conj(I_b) + V_c * conj(I_c);

I_P = 100/N;
I_B = I_P * exp(-j*5*pi/6);
I_C = I_P * exp(j*5*pi/6);

S_P = V_B * conj(I_B) + V_C * conj(I_C);

fprintf('Problem 6.6\n')
fprintf('Turns Ratio = %g\n', N)
fprintf('Primary Power = %g + j %g\n', real(S_P), imag(S_P))
fprintf('Secondary Powr = %g + j %g\n', real(S_s), imag(S_s))
------------------
% 6.061 Problem Set 5, Problem 4 (6.7 from Text)

% This is mostly book-keeping

Vp = 4160;           % primary side, line-line
Vs = 480;            % secondary side, line-line
Vsn = Vs/sqrt(3);    % secondary side, line-neutral
Ra = 50;             % three phase resistors
Rb = 30;
Rc = 30;

N = Vp/Vsn;          % turns ratio: delta-wye
a = exp(j*2*pi/3);   % 120 degree rotation

% Preliminary: Primary Side
Vpn = Vp/sqrt(3);
VA = Vpn * exp(-j*pi/6);   % connection rotates +30deg
VB = VA*a^2;
VC = VA*a;

% part a: secondary side grounded

Va = Vsn;            % set 'real' to the secondary side
Vb = Vsn*a^2;
Vc = Vsn*a;
```

```
Ia = Va/Ra;
Ib = Vb/Rb;
Ic = Vc/Rc;

SS = Va*conj(Ia)+Vb*conj(Ib)+Vc*conj(Ic);

% translate to primary side:

IA = (Ia-Ic)/N;
IB = (Ib-Ia)/N;
IC = (Ic-Ib)/N;

SA = VA*conj(IA);
SB = VB*conj(IB);
SC = VC*conj(IC);
SP = SA + SB + SC;

% output
fprintf('Part A: Secondary Side Grounded\n')
fprintf('Secondary Currents\n')
fprintf('Ia = %g + j %g = %g angle %g\n', real(Ia), imag(Ia), abs(Ia), angle(Ia))
fprintf('Ib = %g + j %g = %g angle %g\n', real(Ib), imag(Ib), abs(Ib), angle(Ib))
fprintf('Ic = %g + j %g = %g angle %g\n', real(Ic), imag(Ic), abs(Ic), angle(Ic))
fprintf('Primary Currents\n')
fprintf('IA = %g + j %g = %g angle %g\n', real(IA), imag(IA), abs(IA), angle(IA))
fprintf('IB = %g + j %g = %g angle %g\n', real(IB), imag(IB), abs(IB), angle(IB))
fprintf('IC = %g + j %g = %g angle %g\n', real(IC), imag(IC), abs(IC), angle(IC))
fprintf('Check: Primary P = %10.0f Q = %10.0f Secondary P = %10.0f Q = %10.0f\n', real(SP), ima

% Part B: secondary side ungrounded

Rab = (Ra*Rb + Ra*Rc + Rb*Rc)/Rc;   % find equivalent delta
Rbc = (Ra*Rb + Ra*Rc + Rb*Rc)/Ra;
Rca = (Ra*Rb + Ra*Rc + Rb*Rc)/Rb;

fprintf('Part B: Secondary Side Ungrounded\n')
fprintf('Rab = %g  Rbc = %g  Rca = %g\n', Rab, Rbc, Rca)

Iab = (Va - Vb)/Rab;                % currents in the delta
Ibc = (Vb - Vc)/Rbc;
Ica = (Vc - Va)/Rca;

Ia = Iab - Ica;                     % now get currents in the wye
Ib = Ibc - Iab;
```

```
Ic = Ica - Ibc;

% repeat the rest of part a: it is now the same

SS = Va*conj(Ia)+Vb*conj(Ib)+Vc*conj(Ic);

% translate to primary side:

IA = (Ia-Ic)/N;
IB = (Ib-Ia)/N;
IC = (Ic-Ib)/N;

SA = VA*conj(IA);
SB = VB*conj(IB);
SC = VC*conj(IC);
SP = SA + SB + SC;

fprintf('Secondary Currents\n')
fprintf('Ia = %g + j %g = %g angle %g\n', real(Ia), imag(Ia), abs(Ia), angle(Ia))
fprintf('Ib = %g + j %g = %g angle %g\n', real(Ib), imag(Ib), abs(Ib), angle(Ib))
fprintf('Ic = %g + j %g = %g angle %g\n', real(Ic), imag(Ic), abs(Ic), angle(Ic))
fprintf('Primary Currents\n')
fprintf('IA = %g + j %g = %g angle %g\n', real(IA), imag(IA), abs(IA), angle(IA))
fprintf('IB = %g + j %g = %g angle %g\n', real(IB), imag(IB), abs(IB), angle(IB))
fprintf('IC = %g + j %g = %g angle %g\n', real(IC), imag(IC), abs(IC), angle(IC))
fprintf('Check: Primary P = %10.0f Q = %10.0f Secondary P = %10.0f Q = %10.0f\n', real(SP), ima

% Part C: Secondary grounded through a resistor
Rn = 10;

RM = [Ra+Rn Rn Rn;Rn Rb+Rn Rn;Rn Rn Rc+Rn];     % matrix of resistances
Vv = [Va; Vb; Vc;];                             % vector of voltages
Is = RM\Vv;                                     % these should be the currents

Ia = Is(1);
Ib = Is(2);
Ic = Is(3);

% repeat the rest of part a: it is now the same

SS = Va*conj(Ia)+Vb*conj(Ib)+Vc*conj(Ic);

% translate to primary side:

IA = (Ia-Ic)/N;
```

```
IB = (Ib-Ia)/N;
IC = (Ic-Ib)/N;

SA = VA*conj(IA);
SB = VB*conj(IB);
SC = VC*conj(IC);
SP = SA + SB + SC;

fprintf('Part C: secondary grounded through a resistor\n')
fprintf('Secondary Currents\n')
fprintf('Ia = %g + j %g = %g angle %g\n', real(Ia), imag(Ia), abs(Ia), angle(Ia))
fprintf('Ib = %g + j %g = %g angle %g\n', real(Ib), imag(Ib), abs(Ib), angle(Ib))
fprintf('Ic = %g + j %g = %g angle %g\n', real(Ic), imag(Ic), abs(Ic), angle(Ic))
fprintf('Primary Currents\n')
fprintf('IA = %g + j %g = %g angle %g\n', real(IA), imag(IA), abs(IA), angle(IA))
fprintf('IB = %g + j %g = %g angle %g\n', real(IB), imag(IB), abs(IB), angle(IB))
fprintf('IC = %g + j %g = %g angle %g\n', real(IC), imag(IC), abs(IC), angle(IC))
fprintf('Check: Primary P = %10.0f Q = %10.0f Secondary P = %10.0f Q = %10.0f\n', real(SP), ima
```