Massachusetts Institute of Technology
Department of Electrical Engineering and Computer Science
6.061 Introduction to Power Systems

Problem Set 2 Solutions   February 15, 2007

Problem 1: Problem 2.7 of the text

The resistance and reactance are in parallel, so:

$$
\begin{aligned}
I_R &= \frac{V_s}{R} = \frac{120}{10} = 12 \\
I_X &= \frac{V_s}{jX} = \frac{120}{j20} = -j6
\end{aligned}
$$

A phasor diagram that shows this is in Figure 1

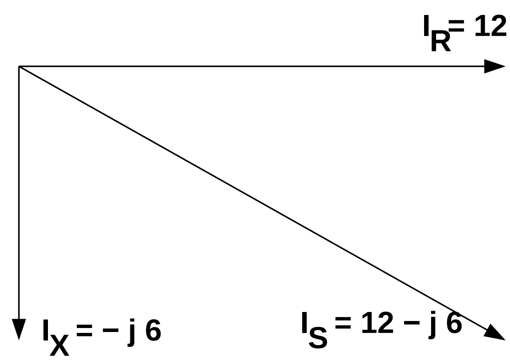

Figure 1: Solution to Problem 7

Real and reactive power are:

$$P + jQ = VI^* = 1440 + j720$$

Problem 2: Problem 2.10 of the text

The two phasor diagrams are shown in Figure 2

Source voltage is:

$$\underline{V} = V_s + jX\underline{I}$$

The locus of this voltage, with arbitrary phase angle of $\underline{I}$ is shown in Figure 3.

And the range of source voltage magnitudes is:

$$90 < |V| < 110$$

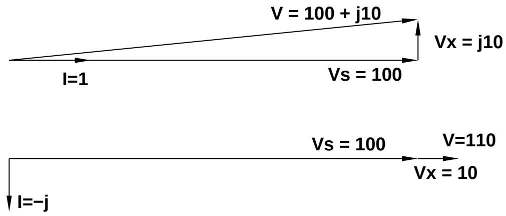

Figure 2: Phasor Diagrams for Problem 10

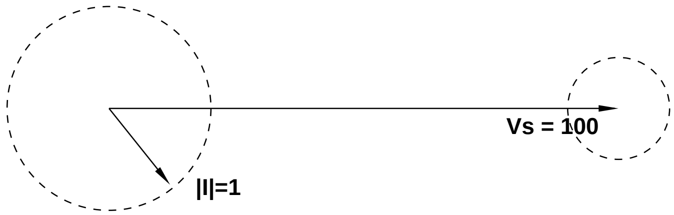

Figure 3: Locus of Current and Voltage Phasors

Problem 3: 1. The switch is *opened* at time $t = 0$, and the current source has a constant current $I_0 = 1A$. The *particular* solution is $V_p = 500$. Noting that the homogeneous equation is:

$$RC\frac{dv_0}{dt} + v_0 = 0$$

which is solved by:

$$v_o = V_H e^{-\frac{t}{RC}}$$

The initial condition is simply:

$$V_o(t = 0+) = 0 = 500 + V_H$$

Noting that $RC = 20mS$, the total solution is

$$v_o = 500\left(1 - e^{-50t}\right)$$

2. The source is changed to be $i_s = \cos\omega t$ with $\omega = 2\pi \times 60Hz$. The *particular* solution is the sinusoidal steady state solution to

$$v_o = \mathrm{Re}\left\{\frac{R}{1 + j\omega RC}e^{j\omega t}\right\}$$

which has magnitude:

$$|V_o| = V_s\frac{R}{\sqrt{1 + (\omega RC)^2}}$$

and phase angle

$$\theta = -\tan^{-1}\omega RC$$

So the final answer for the particular solution is:

$$v_{op} = |V_o| \cos(\omega t + \theta)$$

Because the initial condition for voltage is $v_o(t = 0) = 0$, we must add to the solution a homogeneous part:

$$v_o = |V_o| \cos(\omega t + \theta) - |V_o| \cos\theta e^{-\frac{t}{RC}}$$

Here, noting that $\omega RC = 377 \times 500 \times 40 \times 10^{-6} \approx 7.54$,

$$|V_o| = \frac{500}{\sqrt{1 + 7.54^2}} \approx 65.74$$

$$\theta = -\tan^{-1} 7.54 \approx 1.44\text{radians}$$

This is plotted in Figure 5

3. **For 6.690** The scripts which do the calculation and simulation are appended to the end of this document. They compute a solution that agrees with the analytical solution computed above.

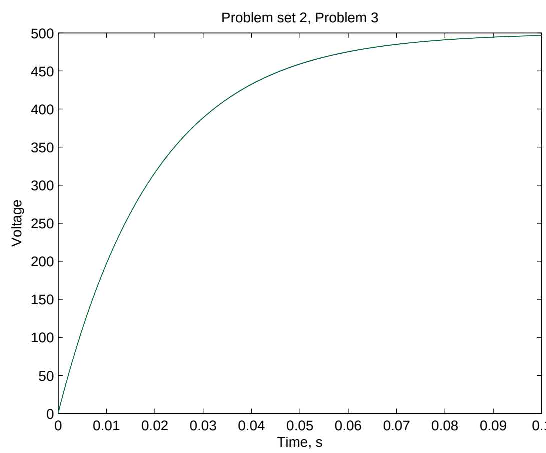

Figure 4: Solution to Problem 3, part 1

Problem 4: 'Buck converter'

1. The *average* output voltage is simply

$$< v_o >= \alpha V_s$$

whera α is the duty cycle

2. We know the maximum current (in steady state) is

$$i_m = \frac{V_s}{R} \frac{1 - e^{-\frac{R}{L}\alpha T}}{1 - e^{-\frac{R}{L}T}}$$

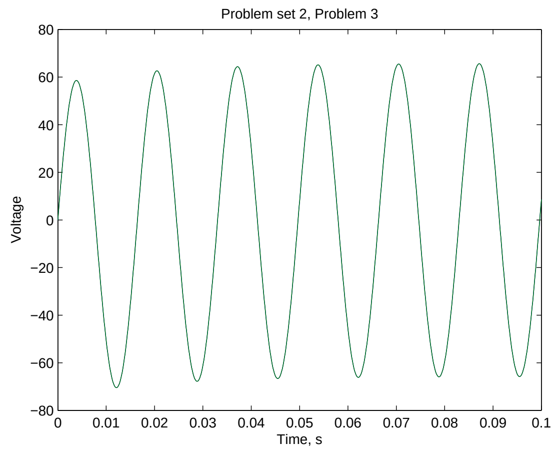

Figure 5: Solution to Problem 3, part 2

where $t_{on} = dT$ and the minimum current is:

$$i_\ell = i_m e^{-\frac{R}{L}(1-\alpha)T}$$

A script which estimates and plots the ripple over a range of duty cycle is in the appendix. Here is the output for 50% duty cycle:

```
-----------------------
50 percent duty cycle
Max Voltage = 25.1562
Min Voltage = 24.8438
Ripple Voltage = 0.312496
```

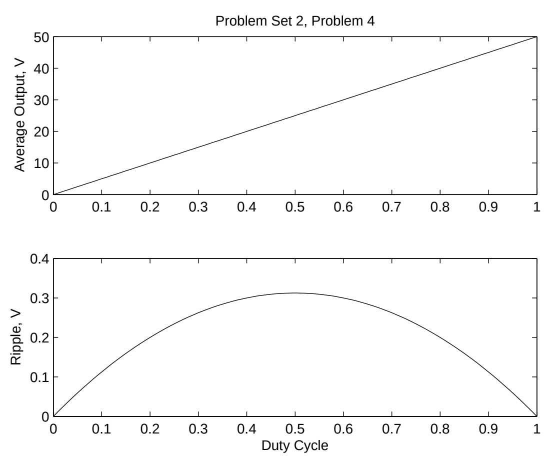

Figure 6: Buck Converter Ripple vs. duty cycle

3. Simulation: A script which does the simulation is located in the Appendix. The simulation uses two scripts to generate the time derivative of current: one for the 'on' switch state, the other for the 'off' switch state. These are repeated in a 'for loop'. The initial condition for each state is simply the final condition for the preceeding state. This script repeats a few cycles at the end to get a better idea of steady state operation. These may be compared with the numbers obtained above.

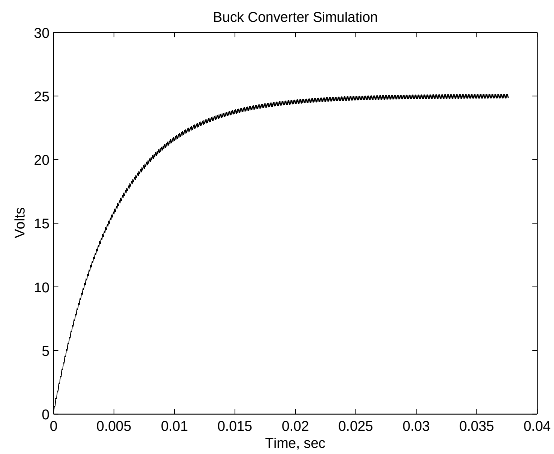

Figure 7: Buck Converter Voltage Buildup

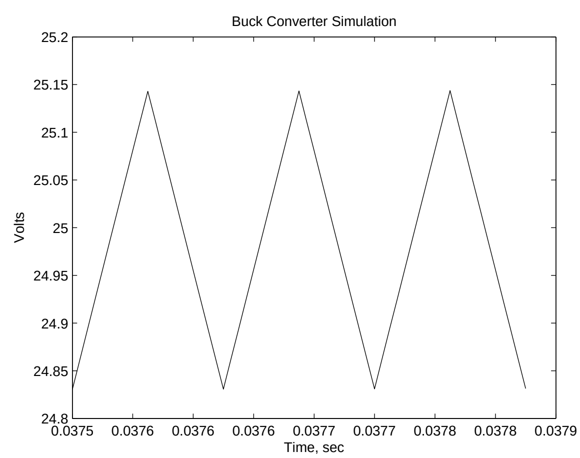

Figure 8: Buck Converter in Steady State

Scripts

```
% 6.061 sp11 Problem Set 2, Problem 3

I=1;
R=500;
C=40e-6;
t = 0:.0001:.1;
omega = 120*pi;

%part 1
% classical solution
vc =R*I .* (1 - exp(-t ./ (R*C)));

% simulation
[ts, vs] = ode23('rc', t, 0);

figure(1)
plot(t, vc, t, vs)
title('Problem set 2, Problem 3')
ylabel('Voltage')
xlabel('Time, s')

% part 2: Driven by a cosine

angle = atan(omega*R*C);
vc = (R/sqrt(1+(omega*R*C)^2)) .* (cos(omega .* t - angle)...
    -cos(angle) .* exp(-t ./ (R*C)));

% simulation
[ts, vs] = ode23('rc2', t, 0);

figure(2)
plot(t, vc, t, vs)
title('Problem set 2, Problem 3')
ylabel('Voltage')
xlabel('Time, s')
------------------
function dv = rc(t, v)
R=500;
C=40e-6;
I=1;
dv=I/C-v/(R*C);
------------------
function dv = rc2(t, v)
```

```
R=500;
C=40e-6;
omega = 120*pi;
I=cos(omega*t);
dv=I/C-v/(R*C);
------------------

\noindent Problem 4: Analytical Calculation of Ripple
% Problem 4: Buck Converter Example

R = 4;
L = .02;
T = 1.25e-4;
dc = .5;
ton = T*dc;
toff = T*(1-dc);
Vs = 50;
Vm =  Vs *(1-exp(-(R/L)*ton))/(1-exp(-(R/L)*T));
Vl = Vm * exp(-(R/L)*toff);
Vr = Vm-Vl;
fprintf('50 percent duty cycle\n');
fprintf('Max Voltage = %g\n',Vm);
fprintf('Min Voltage = %g\n',Vl);
fprintf('Ripple Voltage = %g\n',Vr);
d = 0:.01:1;
t_on = T .* d;
t_off = T .* (1-d);
Vm =  Vs .*(1-exp(-(R/L) .*t_on)) ./(1-exp(-(R/L)*T));
Vl = Vm .* exp(-(R/L) .*t_off);
Vr = Vm-Vl;
Vavg = Vs .* d;
figure(5)
subplot 211
plot(d, Vavg, 'k')
title('Problem Set 2, Problem 4')
ylabel('Average Output, V')
subplot 212
plot(d, Vr,  'k')
ylabel('Ripple, V');
xlabel('Duty Cycle')

Problem 2 Simulation Script:
% 6.061 sp11 Problem set 2, Problem 4 simulation
% buck converter
global Vs L R
```

```
Vs=50;
L=.02;
R=4;
il=[];
t = [];
T = 1.25e-4;         % 8 kHz
d = .5;
ton = T*d;
toff = T*(1-d);
S0=0;
for n = 0:300
    [tt, S] = ode23('bon', [n*T n*T+ton], S0);
    t = [t' tt']';
    il = [il S'];
    S0 = S(length(tt));
    [tt, S] = ode23('boff', [n*T+toff (n+1)*T], S0);
    t = [t' tt']';
    il = [il S'];
    S0 = S(length(tt));
end
vo = R .* il;
figure(3)
clf
plot(t, vo,  'k')
title('Buck Converter Simulation')
ylabel('Volts');
xlabel('Time, sec');
% now just to get the last few cycles
tf =[];
ilf = [];
for n = 300:302
    [tt, S] = ode23('bon', [n*T n*T+ton], S0);
    tf = [tf' tt']';
    ilf = [ilf S'];
    S0 = S(length(tt));
    [tt, S] = ode23('boff', [n*T+toff (n+1)*T], S0);
    tf = [tf' tt']';
    ilf = [ilf S'];
    S0 = S(length(tt));
end
vof = R*ilf;
figure(4)
clf
plot(tf, vof,  'k')
title('Buck Converter Simulation')
```

```
ylabel('Volts');
xlabel('Time, sec');
--------------
function DS = bon(t, il)
global Vs L R
DS = (Vs - R*il)/L;
--------------
function DS = boff(t, il)
global Vs L R
DS = (- R*il)/L;
```

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