Massachusetts Institute of Technology

Department of Electrical Engineering and Computer Science

6.061/6.690 Introduction to Power Systems

Solution To Problem Set 1 February 3, 2011

Problem 1: Simple Problems from the book

1. $240v \times 50A = 12kW$
   $12kW \times 3,414\text{BTU/kWh} = 40,968\text{BTU/h}$
2. $R = \frac{3,414}{.5} = 6,828\text{BTU/kWh}$
3. Assume Coal energy content is 30,870 BTU/kg.
   If R=11,000 BTU/kWh, then coal consumption is:

$$\dot{m} = \frac{11,000\text{BTU/kWh}}{30,870\text{BTU/kg}} \approx 0.3563\text{kg/kWh}$$

   Then, if $P = 1000MW = 10^6 kW$,

$$\dot{M} = 10^6 kW \times .3563 = 3.563 \times 10^5\text{kg/h}$$

$$\times 365.25 \times 24 = 3.12 \times 10^9\text{kg/yr} = 3.12 \times 10^6\text{Tonnes/yr}$$

4. If $R = 30,890\text{BTU/kWh}$,

$$\begin{aligned} \dot{m} &= \frac{9,500\text{BTU/lWh}}{30,890\text{BTU/kg}} \approx .3075\text{kg/kWh} \\ \times 2.959\text{kg } CO_2 \text{ /kg fuel} &= 0.9\text{kg } CO_2\text{/kWh} \\ \times 600,000 \times 24 \times 365.25 &= 4.79 \times 10^9\text{kg } CO_2\text{/yr} = 4.79 \times 10^6\text{T } CO_2\text{/yr} \end{aligned}$$

Problem 2: This problem is solved by superposition: Consider one source at a time and add the solutions.

$$V_{\text{OC}} = 1 \times 8||8 + 10 \times \frac{8}{8+8} = 4 + 5 = 9$$

$$I_{\text{sc}} = 1 \times \frac{4}{4 + 8||8} + 10 \times \frac{8||4}{8 + 8||4} \times \frac{1}{4} = .5 + \frac{5}{8} = \frac{9}{8}$$

Note that $R_{\text{th}} = 4 + 8||8 = 8$ and this confirms the previous answer: $I_{\text{sc}} = \frac{V_{\text{OC}}}{R_{\text{th}}}$

Then:

$$V = V_{\text{OC}} \times \frac{8}{8+8} = 9 \times \frac{1}{2} = 4.5$$

Problem 3: There is a handy-dandy formula in Chapter 4 for this problem, expression 4.12.

$$\begin{aligned} R_1 &= \frac{5 \times 10}{5 + 10_1 0} = 2 \\ R_2 &= \frac{10 \times 10}{5 + 10 + 10} = 4 \end{aligned}$$

But this one can be done easily using more ad-hoc methods. Note driving point impedance is:

$$R_1 + R_2 = 10||15 = 6$$

Then the voltage divider ratio is:

$$\frac{R_2}{R_1 + R_2} = \frac{10}{15} = \frac{2}{3}$$

From which it is straightforward to conclude that $R_2 = 6 \times 2/3 = 4$ and $R_1 = 6 - R_2 = 2$.

Problem 4: The bridge can be characterized as a thevenin equivalent circuit. Without the horizontal resistor of value one, the open circuit voltage will be:

$$V_{\text{OC}} = 18 \times \left(\frac{4}{5} - \frac{1}{5} \right) = 18 \times \frac{3}{5}$$

The Thevenin equivalent resistance is $R_{\text{th}} = 2 \times 1||4 = \frac{8}{5}$. So then, with the resistor of value one is in place:

$$V_o = 18 \times \frac{3}{5} \times \frac{1}{1 + \frac{8}{5}} = \frac{54}{13}$$

Problem 5: This is approximately the picture you should get.

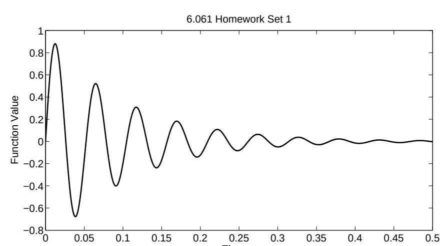

Figure 1: Answer for Problem 5

Problem 6: For 6.690 only The key to doing this easily is shown in Figure 2, where one of the cells of the ladder is called out. It is easy to see that the driving point impedance of that cell of the ladder is simply $2R||2R = R$ and the transfer relationship between input and output is $V = \frac{1}{2}V_{-1}$. Moving left one cell, it is also clear the the driving point resistance looking into the cell is still just R and the transfer is $V_{-1} = \frac{1}{2}V_{-2}$. This is true of each successive cell until we reach the source. The voltage divider ratio between the source and the node just above the source is $V_n = V_s \times \frac{2R||2R}{2R+2R||2R} = \frac{V_s}{3}$.

In this case, $V = V_1 \times \frac{1}{3} \times 2^{-6} + V_2 \times \frac{1}{3} \times 2^{-5}$ if $V_1 = V_2 = 5$, this becomes: $V = \frac{5}{3} \times \left(\frac{1}{32} + \frac{1}{64}\right) = \frac{5}{3} \times \frac{3}{2} \times \frac{1}{32} = \frac{5}{64} \approx .078125$

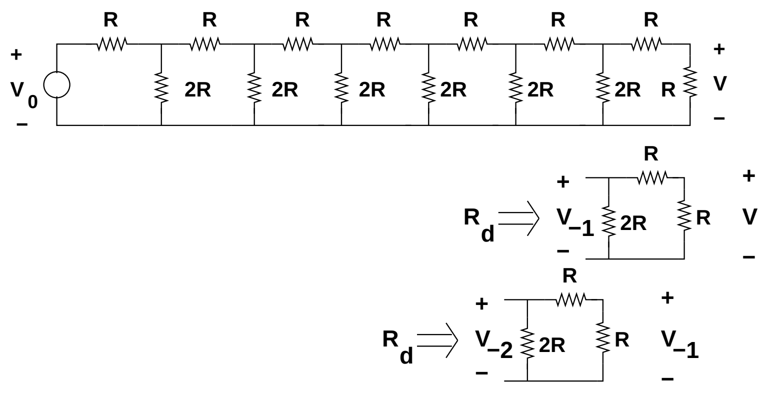

Figure 2: Magic Ladder Circuit

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